

Emmaville Primary School

A Whole-School Approach to Mental and Written Calculations

A Guide for Teachers and Parents



Nursery and Reception – The Foundation Stage

Nursery and Reception are organised to promote social skills and develop understanding of mathematics through games, rhymes, songs, stories, construction, imaginative play, outdoor play, cooking, shopping, music and art, exploring patterns and number in our environment and of course daily routines.

Many routines and activities at home have potential for developing children's mathematical understanding. A question or a comment that is offered during play activities or daily routines can help children see the mathematics they are using and challenge them to think mathematically.

The following ideas are starting points for you to dip into, experiment with and build on.

Theme	Questions/Ideas	Mathematics
Money and Shopping	How much do you think will be in here? Let's count it and see...	Prediction based on experience
	What's this coin? What's this one? Which is worth more? So why is this one smaller?	Recognition of coins
	What coin do you think we've got most of? <i>Make piles of coins (same value) Children will often think the tallest tower has the larger amount of coins as they will disregard the thickness of the coins</i>	Prediction
	When counting, ask your child to watch carefully, and only count when a coin is picked up or placed down. This helps children match a number name to an object when counting	Counting and one-to-one matching
	How much is this? Too much? Too little? How much change do we need?	Calculation and solving problems
The Order of the Day	Collect small amounts of money in a piggy bank, tin, or play till. Children can keep a record of how many of each coin they have (start with just 1p and 2p coins)	Recognition of coins Counting Checking
	<i>What do you do first when you get to school?</i> (eg. Taking coats off, going to the toilet, drawing/painting, washing hands, looking at books) <i>Can you be more detailed?</i> (eg. Open the door, hang my bag up, go to the toilet, wash my hands, sit in the book corner...)	Order Sequence of events
Dates, the Calendar and the Weather	Link above to any home routines... <i>In a few minutes we have to do something else. What do you think we're going to do next? Are you sure? How do you know? What happens after that?</i>	Prediction based on experience Justification
	<i>What day of the week is it? What day was it yesterday? What day will it be tomorrow? How many days until...? How many days in a week? How did you know? Are there always seven days in a week, four weeks in a month, twelve months in a year? What's the number in the date today? Try writing it in the air, on your hand, on the carpet, on someone's back...</i>	The structure of days, weeks and months, passage of time Writing numerals
	Draw a simple chart so that your child can record the weather over a seven day period. A piece of paper with the days of the week written along the bottom or down the left hand side can be used. Your child can then draw a picture or pictures next to each day to show what the weather is like e.g. sun, cloud	Using a chart Using symbols

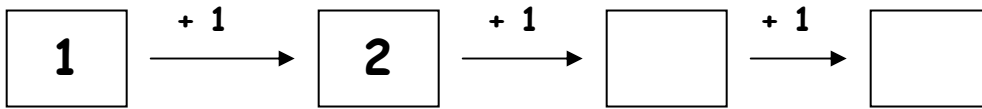
Theme	Questions/Ideas	Mathematics
Birthdays	<i>How old are you?</i> <i>How old were you last year?</i> <i>How old will you be on your next birthday?</i>	Passage of time
	<i>How old is your sister/brother/friend?</i> <i>Can you ever be older/younger than them? Why?</i>	Mathematical reasoning
	Count the number of candles on a cake during a birthday party. Ask the children to clap the same number of times	Counting
News Time	Show your child an item such as a doll, toy car or an interesting picture. After a short time hide the item and ask your child some simple questions about what they might have noticed, e.g. <i>what colour were the eyes? How many wheels were there? What number was on the bus? What was under the table?</i> Some children find this quite difficult. As a lead into the above activity, it is sometimes better to talk to your child about the item before hiding it from view.	Counting Observing and remembering numbers, shapes and colours Using mathematical language Searching for properties
Getting Changed	<i>How long does it take you to get dressed for school?</i> <i>How long does it take you to get ready for bed?</i>	Timing (in minutes or counting)
Helping at Home	Let your child help you to prepare for a meal or a party. <i>How many cups? How many plates? How many sandwiches if everyone has two?</i> Mixing soft drinks e.g. one part squash to 5 parts water	Calculating and checking
	Sorting the washing into colours and whites <i>How many of each? How many scoops of washing powder/how many tablets?</i>	Separate a given number of objects into groups Counting
	Ask your child to help you put away the weekly shopping. What goes in the fridge, which shelf? Where does the washing powder go? What about the bread? Can the items be stored neatly?	Mathematical language (position, size, shape) Counting
In the Street	Encourage your child to look for and say numbers around them, e.g. on houses, shops, buses, car number plates, lifts, speed signs Look for situations where you can encourage your child to count, e.g. Up and down steps, the number of red cars, the number of dogs you see on your trip Encourage your child to look for, and name, shapes they see around them e.g. road signs, on buildings	Recognise and name numerals Counting Recognising and naming shapes
Games	Games can be both fun and educational. For example, hopscotch, skittles or snakes and ladders provide opportunities for children to count and calculate	

Recording calculation in Reception

It is important to emphasise that recording numerals in Reception is only an objective in term 3. Prior to this the emphasis is on recognising and ordering numerals. Teachers **do** need to model simple addition and subtraction using formal notation, including use of operation and equals symbols. Children

should be encouraged to use their own pictorial recording to represent quantities and the results of simple calculations.

When children are ready to use numerals the following are possible ways to record simple calculations:



$$4 + 2 = \boxed{}$$

$$\boxed{\begin{smallmatrix} \cdot \\ \cdot \end{smallmatrix}} + \boxed{\dots} = \boxed{}$$

4	...
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$$4 + \boxed{} = \boxed{}$$

Take away two

5p	+	1p		1p		1p	=	p
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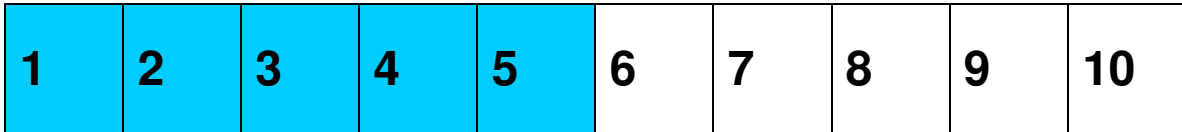


4

Take away 2

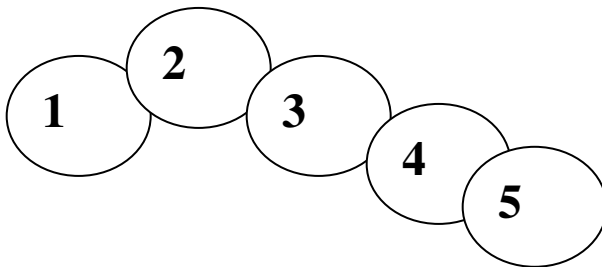


Using number tracks



$$5 + 1 =$$

Adding one



$$3 + 1 =$$

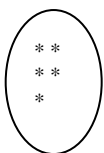
+

=

Year One

Addition

$$5 + 3 = 8$$

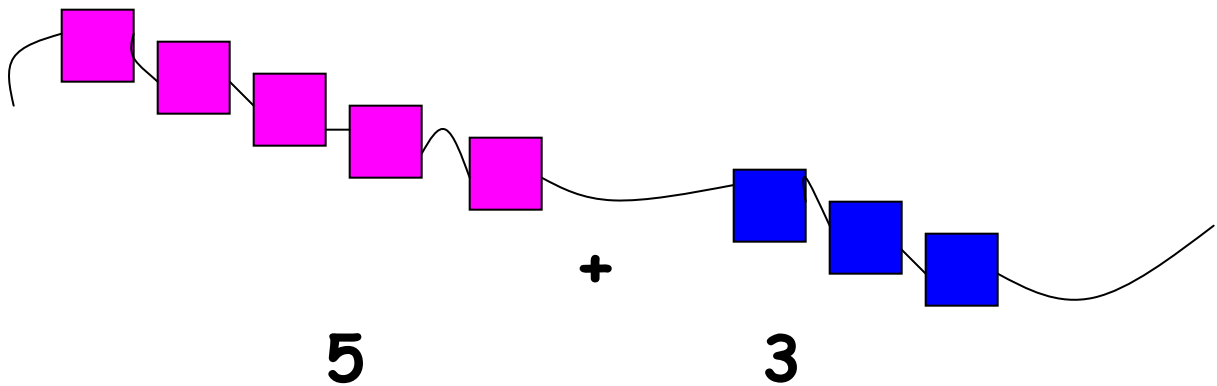
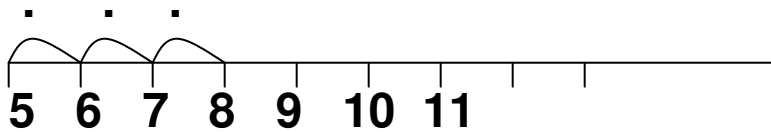


+



$$= 8$$

$$5 + 3$$



Mental Calculation Strategies (+ and -)

Recording Method

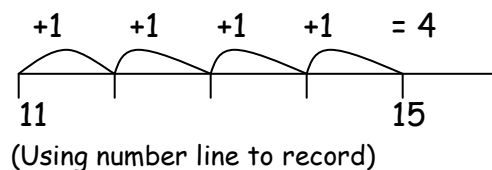
- Use knowledge that addition can be done in any order

$$4 + 7 = 7 + 4 = 11 \text{ (Count on in ones)}$$

- Begin to *partition* into 5 and a bit when adding 6, 7, 8 or 9, then recombine

$$\begin{aligned} 5 + 7 &= 5 + 5 + 2 \\ &= 10 + 2 \\ &= 12 \end{aligned}$$

- Find a small *difference* by counting up



- Identify near doubles

$$\begin{aligned} 6 + 7 &= 6 + 6 + 1 \\ &= 12 + 1 \\ &= 13 \end{aligned} \quad \text{or.....}$$

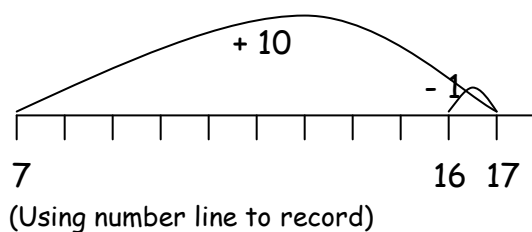
$$6 + 7 = 12 + 1$$

$$= 13$$

- Add 9 to single digit numbers by adding 10 then subtracting 1

$$7 + 9 = 7 + 10 - 1$$

$$17 - 1 = 16 \quad \text{or.....}$$

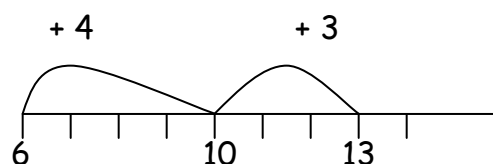


- Begin to bridge through 10, and later 20, when adding a single digit number

$$6 + 7 = 6 + 4 + 3$$

$$= 10 + 3$$

$$= 13 \quad \text{or...}$$



- Use known number facts and place value to add or subtract a pair of numbers mentally

$$2 + 3 = ? \quad 2 + ? = 5$$

$$? + 3 = 5 \quad 7 - 3 =$$

$$7 - ? = 4 \quad ? - 4 = 3$$

Add or subtract a single digit to or from a *teens* number without crossing 10s boundary:

$$14 + 3, 16 + 4, 17 - 6 \text{ etc.}$$

$$15 + 4 = ?, \quad 15 + ? = 19 \text{ etc.}$$

Begin to add a *teens* number to a *teens* number without crossing the 10s boundary:

$$14 + 12, 17 + 12, 13 + 15 \text{ etc.}$$

$$14 + 13 = ?, \quad 14 + ? = 27 \text{ etc.}$$

Year Two

Mental Calculation Strategies (+ and -)

- Use knowledge that addition can be done in any order to do mental calculations more efficiently

For example, putting the larger number first and counting on:

$$5 + 38 = 38 + 5 \text{ (count on 5 to get to 43)}$$

Or use knowledge of 'bonds' to 10 to count on in steps:

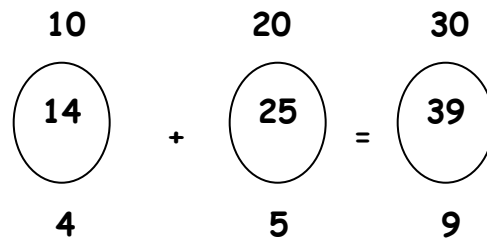
$$\begin{aligned} 6 + 28 &= 6 + 4 + 24 \\ &= 10 + 24 \\ &= 34 \end{aligned}$$

Use knowledge of bonds to 10 to add
Three numbers:

$$3 + 8 + 7 = 10 + 8 = 18$$

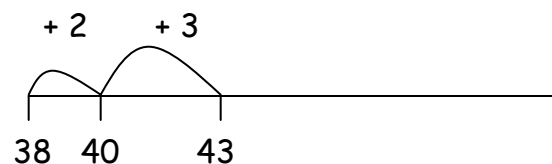
Partition numbers into tens and units:

$$\begin{aligned} 14 + 25 &= (10 + 20) \text{ plus } (4 + 5) \\ &= 30 + 9 \\ &= 39 \end{aligned}$$



- Find a small difference by counting up from the smaller to the larger number

43 - 38 (children could record this using a blank number line)



- Identify near doubles, using doubles already known

$$8 + 9 = 16 + 1 \\ = 17$$

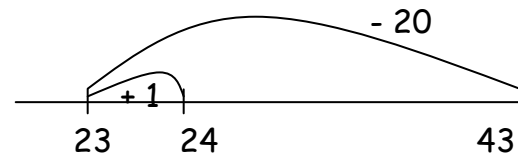
$$18 + 19 = 36 + 1 \\ = 37$$

- Add or subtract 9 or 11 by adding or subtracting 10, then adjusting
Begin to add/subtract 19 or 21

$$23 + 9 = 33 - 1 \\ = 32$$

$$23 - 9 = 13 + 1 \\ = 14$$

$$43 - 19$$



- Use patterns of similar calculations

Develop a pattern such as:

$$\begin{array}{ll} 4 + 6 = 10 & 7 - 4 = 3 \\ 14 + 6 = 20 & 17 - 4 = 13 \\ 24 + 6 = 30 & 27 - 4 = 23 \end{array}$$

and therefore realise that:

$$74 + 6 = 80 \quad 77 - 4 = 73$$

Similarly, recognise and use patterns:

$$\begin{array}{l} 5 + 4 = 9 \\ 50 + 40 = 90 \\ 500 + 400 = 900 \end{array}$$

- State the subtraction corresponding to a given addition, and vice versa

$$13 + 5 = 18 \text{ therefore } 18 - 5$$

- Use known number facts and place value to add/subtract mentally

Answer oral questions such as:

$$35 + 4 \quad 88 - 5$$

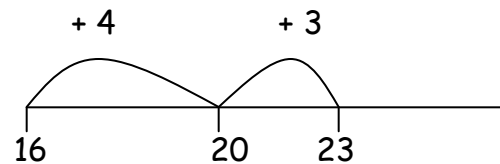
Complete written questions such as:

$$43 + 6 = ? \quad 45 + ? = 48$$

$$67 - 4 = ? \quad ? - 7 = 82$$

- Bridge through 10 or 20, then adjust

$$16 + 7 = 16 + 4 + 3$$



Mental Calculation Strategies (\times and \div)

- Use known number facts and place value to carry out mentally simple multiplications and divisions

Multiply a single digit number by 1 or 10

$$8 \times 1 = ? \quad 1 \times ? = 70$$

$$6 \times 10 = 60 \quad 50 = ? \times 10$$

Divide a two-digit multiple of 10 by 1 or 10

$$70 \div 1 = ? \quad 60 \div ? = 6$$

Begin to double any multiple of 5 up to 50

$$30 \times 2 = ? \quad ? \times 2 = 70$$

Begin to halve any multiple of 10 to 100

$$50 \div 2 = ? \quad ? \div 2 = 15$$

Multiply a single digit number up to 5 by 2, 3, 4 and 5

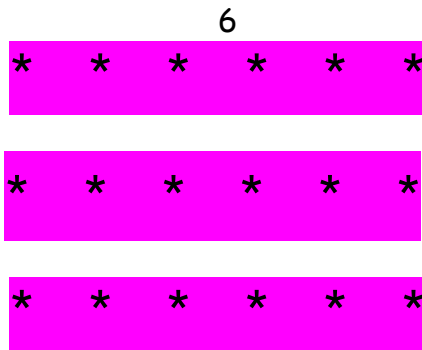
$$7 \times 2 = ? \quad 3 \times ? = 12$$

Understanding Multiplication and Division

Multiplication and division are introduced in year two and teachers use visual models to help children understand how multiplication and division work. Children are taught that multiplication is essentially repeated addition and division is repeated subtraction. At the same time children will be practising

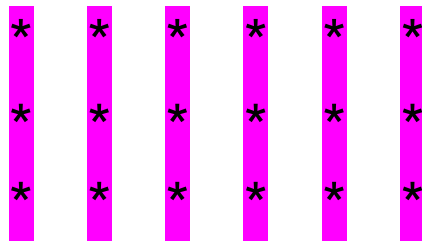
counting forwards and backwards in steps (multiples) and learning multiplication facts from the 2 and 10 times tables. Later in year two pupils will begin to learn facts from the 5 times table.

Dot pattern grids help children understand the relationship between multiplication and division. Seeing numbers set out as rectangular patterns of dots can help children develop an understanding of repeated addition and division as subtraction of groups.



$$3 \times 6 = 18$$

$$\text{or } 18 \div 3 = 6$$



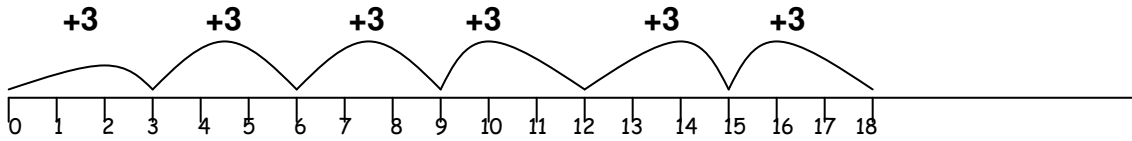
$$6 \times 3 = 18$$

$$\text{or } 18 \div 6 = 3$$

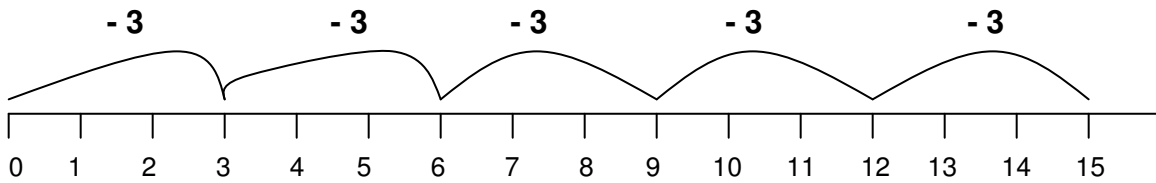
Seeing multiplication and division in this visual way will help children develop a better understanding which will enable them to deal with much more complex calculations in Key Stage 2.

Using a number line is also a very effective way of helping children understand multiplication and division. Children are taught that one way of thinking about multiplication is to see it as lots of repeated addition.

$6 \times 3 = 18$ can be represented on a number line as six jumps of 3 and can also be shown that three jumps of six will give the same answer:



Similarly, $15 \div 3$ can also be represented as repeated subtraction:

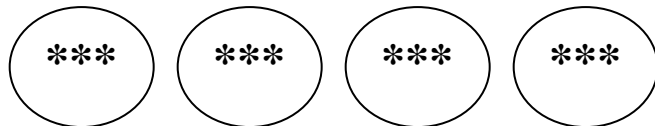
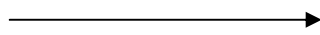


'Sharing' and 'Grouping'

Sharing

If I share 12 sweets between 4 children, how many will they have each?

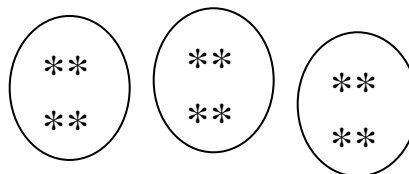
$$12 \div 4 = 3$$



Grouping

How many 4s in 12?

$$12 \div 4 = 3$$



Year Three

Mental Calculation Strategies (+ and -)

- Use knowledge that addition can be done in any order to do mental calculations more efficiently

Put the larger number first and count on

$$7 + 118$$

Count on in ones from 118 or bridge through 10 by adding 2 then 5

$$30 + 64$$

Count on in tens from 64

Add several numbers by using strategies
Such as these:

$$17 + 6 + 3 + 5$$

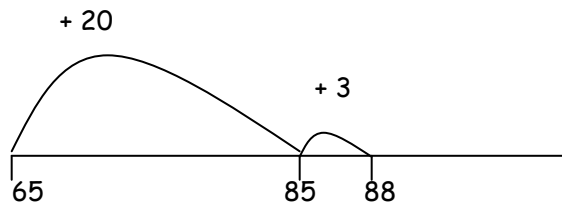
$$20 + 11 = 31$$

$$\begin{aligned} 18 + 5 + 3 + 4 &= 18 + 9 + 3 \\ &= 28 + 2 \\ &= 30 \end{aligned}$$

Partition and recombine

$$\begin{aligned} 65 + 23 &= 80 + 8 \\ &= 88 \quad \text{or....} \end{aligned}$$

$$\begin{array}{ccc} 60 & 20 & 80 \\ \textcircled{65} & + & \textcircled{23} = & \textcircled{88} \\ 5 & 3 & 8 \\ & & \text{or....} \end{array}$$



- Find a small difference by counting up from the smaller to the larger number

This strategy is a good one to use when the two numbers are close together, i.e.
 $73 - 68$

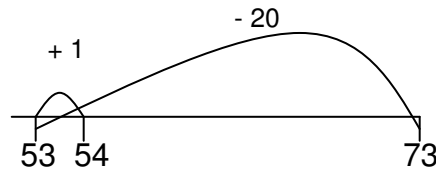
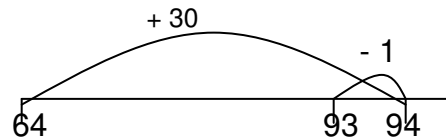
$$68 \xrightarrow{+2} 70 \xrightarrow{+3 \ (+5)} 73$$

- Identify near doubles, using doubles already known

$$\begin{aligned} 45 + 46 &= 90 + 1 \\ 70 + 80 &= 140 + 10 = 150 \\ 17 + 15 &= 30 + 2 = 32 \end{aligned}$$

- Add and subtract mentally a 'near multiple of 10' to or from a two digit number

$$\begin{aligned} 64 + 29 &= 94 - 1 = 93 \\ 73 - 19 &= 53 + 1 = 54 \quad \text{or...} \end{aligned}$$



- Use patterns of similar Calculations

Identify and develop patterns such as:

$$\begin{aligned} 15 + 4 &= 19 & 57 - 4 &= 53 \\ 15 + 14 &= 29 & 57 - 14 &= 43 \\ 15 + 24 &= 39 & 57 - 24 &= 33 \end{aligned}$$

$$\begin{aligned} 3 + 5 &= 8 \\ 30 + 50 &= 80 \\ 300 + 500 &= 800 \end{aligned}$$

- Say or write a subtraction statement corresponding to a given addition statement and vice versa

$$\begin{aligned} 67 + 28 &= 95 \quad \text{therefore} \quad 95 - 28 = 67 \\ & \quad \quad \quad 95 - 67 = 28 \end{aligned}$$

$$86 - 37 = 49 \quad \text{therefore} \quad 49 + 37 = 86$$

- Use known number facts and

Add or subtract a single digit number to or

place value to add/subtract mentally

from *any* three-digit number without crossing the tens boundary

$$567 + 3 \quad 644 + 5 \quad 783 + 6$$

complete questions such as:

$$\begin{array}{l} 608 + ? = 610 \quad 564 + ? = 569 \\ ? - 7 = 872 \quad 768 - ? = 761 \end{array}$$

Add a two-digit number to *any* three digit multiple of 10

$$300 + 67 \quad 400 + 38$$

Subtract a single digit number from *any* three-digit multiple of 10

$$500 - 7 \quad 600 - 9 \quad 300 - 4$$

Add a two-digit number to a multiple of 10, crossing 100

$$70 + 34 \quad 60 + 56 \quad 70 + 88$$

Add or subtract a pair of two-digit numbers without either 10 or 100

$$45 + 33 \quad 62 + 44 \quad 23 + 66$$

- Bridge through a multiple of 10, Then adjust

$$\begin{array}{l} 58 + 7 = 58 + 2 + 5 = 60 + 5 = 65 \\ \text{then refine..} \\ 58 + 7 = 60 + 5 = 65 \end{array}$$

Find a small difference between a pair of numbers that are either side of a multiple of 100

$$605 - 598 \rightarrow 598 \rightarrow 600 \rightarrow 605$$

This uses counting on from the smaller Number - "shopkeeper method"

Begin to add or subtract *any* pair of two-

digit numbers

$$\begin{aligned} 27 + 44 &= 20 + 40 + 7 + 4 \\ &= 60 + 11 \\ &= 71 \quad \text{or....} \end{aligned}$$

$$27 + 44 = 30 + 41 = 71 \quad \text{or....}$$

$$27 + 44 = 30 + 44 - 3$$

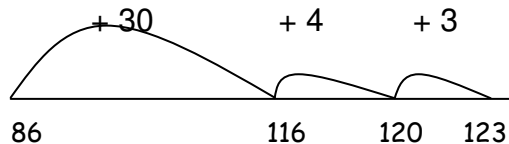
Pencil and paper methods (+ and -)

- Use *informal* pencil and paper methods to support, record and/or explain + and - calculations

Informal jottings can be used as a way of recording and explaining mental methods. Because these jottings illustrate the mental strategies the children are using, they will help pupils remember particular strategies

Counting on in steps of 100, 10 or 1:

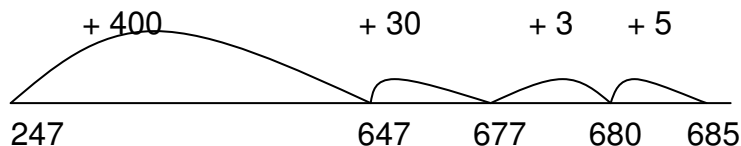
$$86 + 37 = 86 + 30 + 7 = 116 + 7 = 123$$



Here, the 7 has been split into a 4 and 3 to make crossing the tens boundary easier...

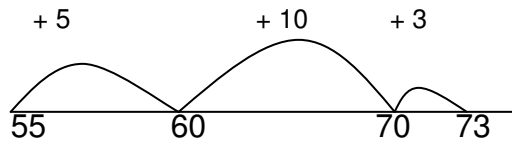
$$\begin{aligned} 247 + 438 &= 247 + 400 + 30 + 8 \\ &= 647 + 30 + 8 \\ &= 677 + 8 \\ &= 685 \end{aligned}$$

..and below, the 8 has been split into a 3 and 5 for the same reason.

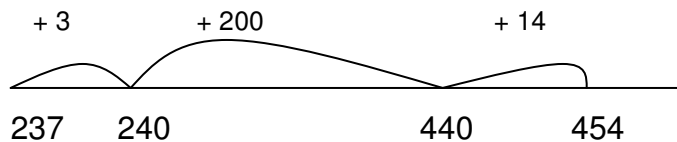


Counting up from the smaller to the larger number when subtracting:

$$73 - 55 \longrightarrow 55 \xrightarrow{+5} 60 \xrightarrow{+10} 70 \xrightarrow{+3} 73 \quad \text{answer: 18}$$

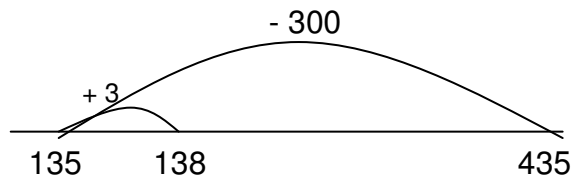


$$454 - 237 \longrightarrow 237 \xrightarrow{+3} 240 \xrightarrow{+200} 440 \xrightarrow{+14} 454 \quad \text{answer: 217}$$



Compensation - taking away too much, then adjusting:

$$435 - 297 = (435 - 300) + 3$$



Some children may be ready for vertical layouts by term 3:

Vertical Layouts for Addition and Subtraction

When children can add or subtract *any* pair of two-digit numbers, recall all their bonds to 20 and partition three-digit numbers, they are ready for vertical calculation methods. However, the first vertical methods are what we call *expanded* methods and they do not involve 'carrying' figures. The idea behind this is that pupils can complete each stage of an expanded method mentally; they are simply arranging each stage vertically instead of horizontally.

Addition

$\begin{array}{r} 68 \\ + 23 \\ \hline 80 \\ + 11 \\ \hline 91 \end{array}$	or.....	$\begin{array}{r} 68 \\ + 23 \\ \hline 11 \\ 80 \\ \hline 91 \end{array}$
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$\begin{array}{r} 127 \\ + 74 \\ \hline 100 \\ 90 \\ + 11 \\ \hline 201 \end{array}$	or....	$\begin{array}{r} 127 \\ + 74 \\ \hline 11 \\ 90 \\ 100 \\ \hline 201 \end{array}$
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Subtraction

Counting on method

Compensation method

$\begin{array}{r} 673 \\ - 285 \\ \hline 5 \text{ (to 290)} \\ 10 \text{ (to 300)} \\ 300 \text{ (to 600)} \\ 73 \text{ (to 673)} \\ \hline 388 \end{array}$	$\begin{array}{r} 673 \\ - 285 \\ \hline 373 \\ + 15 \\ \hline 388 \end{array}$
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Decomposition

$$\begin{array}{r} 72 \\ - 46 \\ \hline \end{array} = \begin{array}{r} 70 + 2 \\ - 40 - 6 \\ \hline \end{array} = \begin{array}{r} 60 + 12 \\ - 40 - 6 \\ \hline \end{array}$$

$$20 + 6 = 26$$

Mental calculation strategies (\times and \div)

- Multiply by 10 and 100 by shifting digits one/two places to the left

Develop patterns such as:

1	2	3	4	5	6	7
10	20	30	40	50	60	70
100	200	300	400	500	600	700

300	400	500...	600	700	800
30	40	50	60	70	80
3	4	5	6	7	8

- Use doubling or halving

Work out 4 times table facts quickly by doubling facts from the 2 times table:

$$8 \times 2 = 16 \quad \text{therefore} \quad 8 \times 4 = 32$$

If $1 \times 24 = 24$ then double 24 (48)

Must be 2×24 .

If $2 \times 24 = 48$, $96 = 4 \times 24$ and so on.

Find quarters by finding a half of a half:

$$\frac{1}{4} \text{ of } 24 = \frac{1}{2} \text{ of } 12 = 6$$

$$\frac{1}{4} \text{ of } 180 = \frac{1}{2} \text{ of } 90 = 45$$

- Say or write a division statement corresponding to a given multiplication statement

$$8 \times 5 = 40 \quad 40 \div 5 = 8$$

$$40 \div 8 = 5$$

Using numbers such as 2, 5 and 10, write or say different multiplication or division 'sentences' i.e.

$$10 \div 5 = 2, \quad 5 \times 2 = 10...$$

- Use known number facts and place value to carry out mentally simple multiplications and divisions

Multiply a single digit by 1, 10 or 100

$$8 \times 10 \quad 9 \times 100 \quad 7 \times ? = 70$$

$$60 \times ? = 60 \quad ? \times 10 = 50$$

Divide a three-digit multiple of 100 by 10 or 100

$$600 \div 10 = ? \quad 400 \div ? = 4$$

Double any multiple of 5 up to 50

$$25 \longrightarrow 50$$

$$\begin{array}{l} 45 \longrightarrow 90 \\ 15 \longrightarrow 30 \end{array}$$

Halve any multiple of 10 to 100

$$\begin{array}{l} 70 \longrightarrow 35 \\ 90 \longrightarrow 45 \\ 30 \longrightarrow 15 \end{array}$$

Multiply a two-digit multiple of 10 up to 50 by 2, 3, 4, 5 or 10

Respond orally to questions such as:

$$30 \times 3 \quad 50 \times 4 \quad 40 \times 5$$

Answer written questions such as:

$$50 \times 2 = ? \quad 30 \times ? = 150$$

$$? \times 10 = 400 \quad 20 \times ? = 80$$

Multiply a two-digit number by 2, 3, 4 or 5 without crossing the tens boundary

$$22 \times 3 = ? \quad 42 \times ? = 168$$

$$28 = 14 \times ? \quad ? \times 5 = 125$$

Year Four

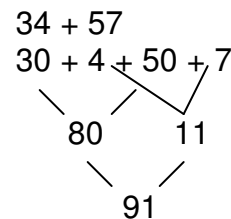
Mental calculation strategies (+ and -)

- Find a small difference by counting Up $83 - 79 \rightarrow 79 + 1 (80) \rightarrow + 3 (83)$
 $3002 - 2993$ is $7 + 2 = 9$

- Count on or back in steps of 1, 10 Or 100 For example, work out mentally:
 $1006 - 8$ (count back in ones from 1006)
 $567 + 40$ (count on in tens from 567)
 $382 - 50$ (count back in tens from 382)
 $240 + 600$ (count on in hundreds...)

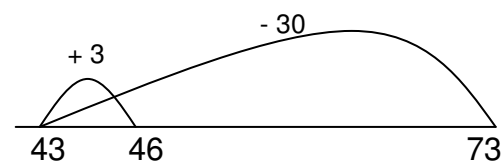
- Partition into tens and units, adding the tens first

$$\begin{aligned} 34 + 57 &= 30 + 50 + 4 + 7 \\ &= 80 + 11 \\ &= 91 \quad \text{or...} \\ 34 + 57 &= 80 + 11 = 91 \quad \text{or..} \end{aligned}$$



- Identify near doubles using known doubles $29 + 28 = 60 - 3 = 57$
 $38 + 36 = 80 - 6 = 74$
 $370 + 380 = 800 - 50 = 750$

- Add or subtract the nearest multiple of 10, then adjust $54 + 28 = 84 - 2 = 82$
 $73 - 27 = 43 + 3 = 46$ or....



- Continue to use the relationship between $47 + 18 = 65$

addition and subtraction

then you also know that:

$$18 + 47 = 65$$

$$65 - 47 = 18 \quad 65 - 18 = 47$$

- Add 3 or 4 small numbers
finding pairs that total 10, 9
or 11...

$$4 + 7 + 9 + 1 = 11 + 10 = 21$$

$$3 + 8 + 9 + 5 = 11 + 10 + 4 = 25$$

$$5 + 7 + 8 + 3 = 10 + 10 + 3 = 23$$

Add three two-digit multiples
of 10

$$30 + 50 + 40 = 90 + 30 = 120 \text{ or..}$$

$$30 + 50 + 40 = 100 + 20 = 120$$

- Use known number facts and
place value to add or subtract
any pair of two-digit whole
numbers

Continue to add or subtract two-digit
multiples of 10

$$40 + 80 \quad 140 - 70$$

$$70 + ? = 120 \quad ? - 40 = 130$$

Add or subtract a pair of multiples of 100,
crossing 1000

$$400 + 800 \quad 1300 - 600$$

$$300 + 800 = ? \quad ? - 400 = 1700$$

Revise adding/subtracting a multiple of
10 to/from a two-digit or three-digit
number without crossing the hundreds
boundary

$$43 + 50 \quad 564 - 50 \quad 62 + ? = 92$$

$$76 - ? = 16 \quad ? + 40 = 593$$

Revise adding a two or three-digit number
to a multiple of 10, 100 or 1000

$$60 + 34 \quad 300 + 107 \quad 2000 + 532$$

$$360 + ? = 386 \quad ? + 56 = 266$$

Find what to add to a two or three-digit
number to make 100 or the next higher
multiple of 100

$$47 + ? = 100 \quad ? + 38 = 100$$

$$245 + ? = 300 \quad ? + 37 = 200$$

Find what to add to a four-digit multiple of
100 to make the next higher multiple of
1000

$$2800 + ? = 3000 \quad ? + 700 = 9000$$

Add a single digit to *any* three or four-
digit number, crossing the tens
boundary

$$587 + 7 \quad 678 + 5$$

$$583 + ? = 591 \quad ? + 6 = 374$$

Subtract a single digit from a multiple of
100 or 1000

$$800 - 8 \quad 4000 - 6$$

$$700 - 5 = ? \quad 400 - ? = 392$$

Subtract a single digit from a three or
four-digit number, crossing the tens
boundary

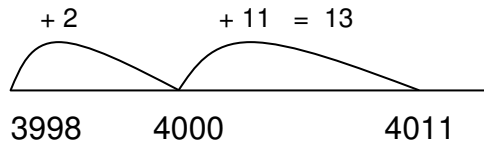
$$514 - 7 \quad 2036 - 8$$

$$422 - 7 = ? \quad 302 - ? = 295$$

$$2104 - ? = 2097 \quad ? - 6 = 1329$$

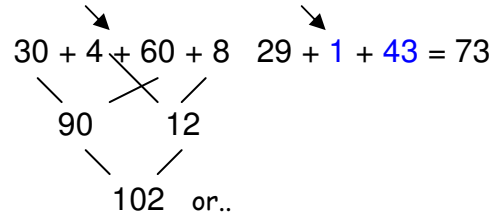
Find a small difference between a pair of numbers lying either side of a multiple of 1000

$$\begin{array}{l} 8004 - 7997 \quad 2001 - 1993 \\ 4011 - 3998 = ? \quad 2004 - ? = 13 \end{array}$$



Add or subtract any pair of two-digit numbers, including crossing the tens boundary

$$\begin{array}{l} 56 + 38 \quad 84 - 27 \\ 34 + 68 = ? \quad 73 - ? = 29 \end{array}$$



$$34 + 68 = 90 + 12 = 102$$

Pencil and paper methods (+ and -)

- Use informal pencil and paper methods to support, record or explain additions and subtractions.
- Develop and refine written methods for:
column addition/subtraction of two whole numbers less than 1000, and addition of more than two such numbers.

Informal written methods

Addition

Adding the most significant digits first

$$\begin{array}{r} 476 \\ + 57 \\ \hline 400 \\ 120 \\ \hline 13 \\ \hline 533 \end{array} \quad \begin{array}{r} 685 \\ + 38 \\ \hline 600 \\ 110 \\ \hline 13 \\ \hline 723 \end{array} \quad \begin{array}{r} 419 \\ + 86 \\ \hline 400 \\ 90 \\ \hline 15 \\ \hline 505 \end{array} \quad \begin{array}{r} 345 \\ + 136 \\ \hline 400 \\ 70 \\ \hline 11 \\ \hline 481 \end{array}$$

Compensation method

$$\begin{array}{r} 658 \\ + 87 \\ \hline 758 \text{ (658 + 100)} \\ - 13 \text{ (take 13 away because we've added 13 too many)} \\ \hline 745 \end{array}$$

Subtraction

Counting up

$$\begin{array}{r} 553 \\ - 76 \\ \hline 480 \\ + 20 \text{ (100)} \\ \hline 400 \text{ (500)} \\ + 53 \text{ (553)} \\ \hline 477 \end{array}$$

or...

adjustment (take too much away, then add back on)

$$\begin{array}{r} 553 \\ - 76 \\ \hline 453 \text{ (553 - 100)} \\ + 24 \text{ (because } 100 - 76 = 24) \\ \hline 477 \end{array}$$

Standard written methods

Addition

Adding the least significant digits, introducing 'carrying' figures

$\begin{array}{r} 446 \\ + 75 \\ \hline 11 \\ 110 \\ \hline 400 \\ \hline 521 \end{array}$	<p>Leading to 'carrying' below the line</p> <p>→</p>	$\begin{array}{r} 543 \\ + 37 \\ \hline 580 \\ 1 \end{array}$	$\begin{array}{r} 688 \\ + 31 \\ \hline 719 \\ 1 \end{array}$	$\begin{array}{r} 375 \\ + 87 \\ \hline 462 \\ 11 \end{array}$
--	--	---	---	--

Subtraction

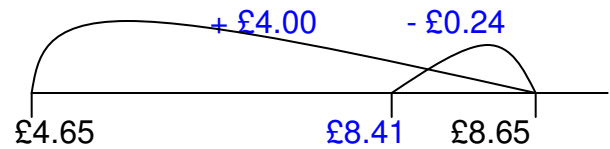
Decomposition

$\begin{array}{r} 753 \\ - 76 \\ \hline \end{array}$	=	$700 + 50 + 3$	leading to....	
		$- 70 + 6$		
	=	$700 + 40 + 13$		$\begin{array}{r} 743 \\ - 76 \\ \hline \end{array}$
		$- 70 + 6$		
	=	$600 + 140 + 13$		$\begin{array}{r} 643 \\ - 76 \\ \hline \end{array}$
		$- 70 + 6$		
		$600 + 70 + 7 = 677$		677

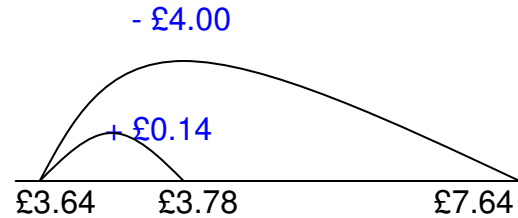
Decimals

Many children will need to use informal methods for adding and subtracting decimals - even in contexts such as money. Methods learnt in year 3 using the blank number line may be more helpful until children are secure in their understanding of the value of digits in decimal numbers.

$$£4.65 + £3.76$$



$$£7.64 - £3.86$$



Mental calculation strategies (\times and \div)

- Use doubling or halving starting from known facts, e.g. to multiply by 4, double and double again, to multiply by 5, multiply by 10 and then halve.

$$12 \times 4 = 24 \times 2 = 48$$

$$12 \times 5 = 120 \div 2 = 60$$

$$13 \times 20 = 130 \times 2 = 260$$

$$64 \div 4 = 32 \div 2 = 16$$

$$120 \div 5 = 12 \times 2 = 24$$

Work out multiples of 15 by doubling:

$$1 \times 15 = 15$$

$$2 \times 15 = 30$$

$$4 \times 15 = 60$$

$$8 \times 15 = 120$$

$$16 \times 15 = 240$$

32 \times 15 = 480...and use combinations to work out..

$$13 \times 15 = 120 + 60 + 15 = 195$$

$$17 \times 15 = 240 + 15 = 255$$

$$\frac{1}{4} \text{ of } 120 \xrightarrow{\frac{1}{2}} 60 \xrightarrow{\frac{1}{2}} 30$$

$$\frac{1}{8} \text{ of } 72 \xrightarrow{\frac{1}{2}} 36 \xrightarrow{\frac{1}{2}} 18 \xrightarrow{\frac{1}{2}} 9$$

- Use closely related facts (e.g. to multiply by 9 or 11, multiply by 10 and adjust; develop the $\times 6$ table from the $\times 4$ and $\times 2$ tables)

$$16 \times 11 = 160 + 16 = 176$$

$$16 \times 9 = 160 - 16 = 144$$

$$7 \times 6 = (7 \times 4) + (7 \times 2)$$

$$= 28 + 14$$

$$= 42$$

□ Partition

$$\begin{aligned} 23 \times 3 &= (20 \times 3) + (3 \times 3) \\ &= 60 + 9 \\ &= 69 \end{aligned}$$

$$117 \times 3 = 300 + 30 + 21 = 351$$

□ Use the relationship between multiplication and division

$$\begin{aligned} 13 \times 8 &= 104 \quad \text{therefore...} \\ 8 \times 13 &= 104 \\ 104 \div 8 &= 13 \\ 104 \div 13 &= 8 \end{aligned}$$

□ Use known number facts and place value to multiply and divide whole numbers, including by 10 and then by 100 (whole number answers)

Multiply a two or three-digit number by 10 or 100

$$\begin{aligned} 234 \times 10 & \quad 47 \times 100 \\ 76 \times 100 &= ? \quad 670 \times ? = 6700 \end{aligned}$$

Divide a four-digit multiple of 1000 by 10 or 100

$$\begin{aligned} 6000 \div 100 & \quad 8000 \div 10 \\ 7000 \div ? &= 70 \quad ? \div 100 = 50 \end{aligned}$$

Double any multiple of 5 up to 100

$$35 \times 2 = ? \quad ? \times 2 = 170$$

Halve any multiple of 10 to 200

$$130 \div 2 = ? \quad ? \div 2 = 75$$

Practise multiplying a two-digit multiple of 10 by 2, 3, 4, 5, or 10 and begin to multiply by 6, 7, 8 or 9

$$\begin{aligned} 30 \times 3 & \quad 50 \times 4 & \quad 20 \times 10 \\ 80 \times ? &= 160 & \quad ? \times 4 = 120 \\ 60 &= 10 \times ? & \quad 140 = ? \times 4 \end{aligned}$$

Pencil and paper methods (\times and \div)

- Approximate first. Use informal pencil and paper methods to support, record or explain multiplications and divisions. develop and refine written methods for $TU \times U$, $TU \div U$

Informal written methods

Multiplication

The grid method - $TU \times U$

33×6 is approximately $30 \times 6 = 180$

$$\begin{array}{r} \text{X} \quad 30 \quad 3 \\ 6 \quad \boxed{180} \quad \boxed{18} \end{array} = 198$$

26×7 is approximately $25 \times 7 = 175$

$$\begin{array}{r} \text{x} \quad 20 \quad 6 \\ 7 \quad \boxed{140} \quad \boxed{42} \end{array} = 182$$

Division

Using multiples of the divisor - $TU \div U$

$$\begin{aligned} 83 \div 5 &= (50 + 30 + 3) \div 5 \\ &= 10 + 6 \text{ remainder } 3 \\ &= 16 \text{ remainder } 3 \end{aligned}$$

Standard written methods

Short multiplication

$$\begin{array}{r} 26 \\ \times 7 \\ \hline 140 \\ \underline{42} \\ 182 \end{array} \quad \xrightarrow{\text{leading to}} \quad \begin{array}{r} 26 \\ \times 7 \\ \hline 182 \\ \text{4} \end{array}$$

Short division

$$\begin{array}{r} 87 \div 6 \\ 6 \overline{) 87} \\ \underline{-60} \quad (10 \times 6) \\ 27 \\ \underline{-24} \quad (4 \times 6) \\ 3 \end{array}$$

Answer: 14 remainder 3

Year Five

Mental Calculation Strategies (+ and -)

- Find differences by counting up through next multiple of 10, 100 or 1000
$$\begin{array}{r} 7004 - 3448 \\ 3448 + 52 \text{ (3500)} \\ + 500 \text{ (4000)} \\ + 3004 \text{ (7004)} \\ \hline 3556 \end{array}$$
- Partition into HTU, adding the most significant digits first
$$\begin{array}{l} 436 + 58 = 400 + 30 + 50 + 6 + 8 \\ = 480 + 14 \\ = 494 \\ 327 - 64 = 327 - 60 - 4 \\ = 267 - 4 \\ = 263 \end{array}$$
- Identify near doubles, such as $1.5 + 1.6$
 $1.5 + 1.6 = 3 + 0.1 = 3.1$
- Add or subtract the nearest Multiple of 10 or 100, then adjust
Add/subtract 9, 19, 29... or 11, 21 etc., by adding/subtracting 10, 20, 30..then adjusting
$$\begin{array}{l} 367 + 39 = 407 - 1 = 406 \\ 273 - 19 = 253 + 1 = 254 \end{array}$$
- Develop further the relationship Between addition and subtraction
$$\begin{array}{l} 146 + 117 = 263, \text{ therefore....} \\ 263 - 146 = 117 \\ 263 - 117 = 146 \end{array}$$

Work out a calculation such as, $13.6 + 8.7$ or $10.5 - 4.8$ and then write or state three other related facts
- Add several numbers (four or five single digits, or multiples of 10)
$$\begin{array}{l} 6 + 9 + 7 + 8 + 3 \\ 30 + 60 + 50 \end{array}$$
- Use known number facts and Place value for mental addition And subtraction
Add or subtract three-digit multiples of 10
$$\begin{array}{l} 480 + 370 \quad 650 - 280 \\ \text{and explain method} \end{array}$$

Add three or more three-digit multiples
Of 100

$$300 + 500 + 400$$

$$200 + ? + 600 = 1100$$

Add/subtract a multiple of 100 to/from
a three or four-digit number, crossing
1000

$$476 + 700$$

$$1763 - 800$$

$$500 + ? = 1744 \quad ? + 355 = 1255$$

and explain method

Add/subtract a three-digit multiple of 10
to/from a three-digit number, without
crossing the hundreds boundary

$$340 + 256 \quad 455 + 500$$

$$620 + 127 = ? \quad 456 + ? = 776$$

$$672 - 340 = ? \quad ? - 236 = 634$$

Continue to find what to add to a three-
digit number to make the next higher
multiple of 100

What must be added to 346 to make 400?

$$348 + ? = 400 \quad ? + 37 = 600$$

Find what to add to a decimal with units
and tenths to make the next higher
whole number

What must be added to 4.7 to make 5?

$$3.4 + ? = 4 \quad ? + 0.2 = 7$$

Find the difference between a pair of
numbers lying either side of a multiple
of 1000

$$4004 - 3897 = 107 \text{ - count up from } 3897.$$

$$\begin{array}{r} +3 \qquad +104 \\ 3897 \rightarrow 3900 \rightarrow 4004 \end{array}$$

Add or subtract a pair of decimal
fractions each with units and tenths,
or with tenths and hundredths,
including crossing the units boundary
or the tenths boundary

$$4.3 + 7.8 \quad 5.3 - 2.8$$

$$5.7 + ? = 10.4 \quad ? - 3.6 = 3.5$$

$$0.55 + 0.72 = ?$$

Pencil and Paper Methods (+ and -)

- Use informal pencil and paper methods to support, record and explain additions and subtractions. **Extend written methods to: column addition/subtraction of two whole numbers less than 10 000;** addition of more than two whole numbers less than 10 000; addition or subtraction of a pair of decimal fractions, both with one or both with two decimal places

Informal Written Methods

Addition

Adding significant digits first

$$\begin{array}{r} 478 \\ + 366 \\ \hline 700 \\ 130 \\ \hline 14 \\ \hline 844 \end{array}$$

$$\begin{array}{r} 5675 \\ + 786 \\ \hline 5000 \\ 1300 \\ \hline 150 \\ 11 \\ \hline 6461 \end{array}$$

add mentally from the top,

when children are confident with this method, they can be taught to add from the right (still using the expanded method):

$$\begin{array}{r} 5675 \\ + 786 \\ \hline 11 \\ 150 \\ 1300 \\ \hline 5000 \\ \hline 6461 \end{array}$$

This method helps children understand what is happening when we introduce 'carrying' figures

Compensation (add too much then adjust)

$$\begin{array}{r} 776 \\ + 288 \\ \hline 1076 \quad (776 + 300) \\ - 12 \\ \hline 1064 \end{array}$$

$$\begin{array}{r} 7.76 \text{ cm} \\ + 2.88 \text{ cm} \\ \hline 10.76 \text{ cm} \\ - 0.12 \text{ cm} \\ \hline 10.64 \text{ cm} \end{array}$$

extend to decimals

$$\begin{array}{r} £56.75 \\ + £7.86 \\ \hline 0.11 \\ 1.50 \\ 13.00 \\ \hline 50.00 \\ \hline £64.61 \end{array}$$

Subtraction

Counting up

$\begin{array}{r} 665 \\ - 278 \\ \hline 22 \quad (300) \\ 300 \quad (600) \\ \hline 65 \quad (665) \\ 387 \end{array}$	leading to →	$\begin{array}{r} 665 \\ - 278 \\ \hline 22(300) \\ 365(665) \\ \hline 387 \end{array}$	$\left. \begin{array}{r} £6.65 \\ - £2.78 \\ \hline £0.22 \quad (£3.00) \\ £3.65 \quad (£6.65) \\ \hline £3.87 \end{array} \right\} \text{extend to decimals}$
<p>Compensation (take away too much then adjust)</p> $\begin{array}{r} 665 \\ - 278 \\ \hline 365 \quad (-300) \\ + 22 \\ \hline 387 \end{array}$	→	$\begin{array}{r} £6.65 \\ - £2.78 \\ \hline £3.65 \quad (-£3.00) \\ + £0.22 \\ \hline £3.87 \end{array}$	

Standard Written Methods

Addition

Using 'carrying'

$\begin{array}{r} 679 \\ + 465 \\ \hline 1144 \\ 11 \end{array}$	$\begin{array}{r} 2547 \\ + 676 \\ \hline 3223 \\ 111 \end{array}$	→	$\begin{array}{r} £5.68 \\ + £4.87 \\ \hline £10.55 \\ 111 \end{array}$
--	--	---	---

Subtraction

Decomposition

Progression from expanded method to compact method should not be rushed.

$$\begin{array}{r} 643 \\ - 87 \\ \hline \end{array}$$

$$\begin{array}{r} 600 + 40 + 3 \\ - 80 + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 600 + 30 + 13 \\ - 80 + 7 \\ \hline \end{array} \xrightarrow{\text{adjust from T to U}} \begin{array}{r} 633 \\ - 87 \\ \hline \end{array}$$

$$\begin{array}{r} 500 + 130 + 13 \\ - 80 + 7 \\ \hline 500 + 50 + 6 = 556 \end{array} \xrightarrow{\text{adjust from H to T}} \begin{array}{r} 533 \\ - 87 \\ \hline 556 \end{array}$$

Mental Calculation Strategies (\times and \div)

- Use doubling or halving, starting from known facts.

$$\text{Double } 76 = 140 + 12 = 152$$

$$\text{Half of } 486 = 200 + 40 + 3 = 243$$

Double a number ending in 5 and halve the other number

$$18 \times 5 = 9 \times 10 = 90$$

$$26 \times 5 = 13 \times 10 = 130$$

$$37 \times 5 = 18.5 \times 10 = 185 \text{ or...}$$

$$18 \times 5 = 180 \div 2 = 90$$

Halve an even number in a calculation then double the answer

$$18 \times 51 = 9 \times 51 \times 2$$

$$= 459 \times 2$$

$$= 918$$

Multiply by 50 by multiplying by 100 then halving

$$47 \times 50 = 4700 \div 2$$

$$= 2350$$

Work out the 16 times table using

Knowledge of 8 times table

$$6 \times 16 = (6 \times 8) \times 2$$

Work out mentally calculations

such as:

$$23 \times 25 \quad 10 \times 25 = 250$$

$$20 \times 25 = 500$$

$$3 \times 25 = 75$$

$$23 \times 25 = 575$$

Explain how to find sixths by halving thirds, quarters by halving eighths etc.

$$\frac{1}{8} \text{ of } 400 \text{ is } 50$$

$$\frac{1}{4} \text{ of } 400 \text{ is } 100 \text{ half of this is } 50$$

- Use factors

$$7 \times 24 = 7 \times 2 \times 3 \times 4$$

$$= 14 \times 3 \times 4$$

$$= 42 \times 4$$

$$= 168$$

- Use closely related facts (e.g. multiply by 19 or 21 by multiplying by 20 and adjusting; develop the x12 table from the x10 and x2 tables

$$\begin{aligned}
 8 \times 12 &= 80 + 16 = 96 \\
 16 \times 19 &= (16 \times 20) - 16 \\
 &= 320 - 16 \\
 &= 304 \\
 16 \times 21 &= (16 \times 20) + 16 \\
 &= 320 + 16 \\
 &= 336
 \end{aligned}$$

- Partition

$$\begin{aligned}
 86 \times 6 &= (80 \times 6) + (6 \times 6) \\
 &= 480 + 36 \\
 &= 516
 \end{aligned}$$

- Use the relationship between multiplication and division

$$\begin{aligned}
 36 \times 3 &= 108 \dots \text{therefore} \\
 108 \div 3 &= 36 \quad 108 \div 36 = 3
 \end{aligned}$$

$$\begin{aligned}
 12 \times 8 &= 96 \dots \text{therefore} \\
 \frac{1}{8} \text{ of } 96 &\text{ is } 12 \\
 \frac{1}{12} \text{ of } 96 &\text{ is } 8
 \end{aligned}$$

- Use known facts and place value
To multiply and divide mentally

Divide a four-digit multiple of 100 by 1000, 100 or 10

$$7400 \div 100 \quad 8700 \div 1000$$

Answer questions such as:

Find one hundredth of 4500

Find one thousandth of 7600

Find one tenth of 9300

$$? \div 100 = 63$$

$$4500 \div ? = 4.5$$

multiply a two-digit multiple of 10 or a three-digit multiple of 100 by a single-digit number

$$600 \times 7 \quad 90 \times 8$$

$$900 \times 10 = ? \quad 60 \times ? = 600$$

Multiply a two-digit multiple of 10 by a three-digit multiple of 100

$$40 \times 300 \quad 60 \times 700$$

$$50 \times ? = 40\,000$$

$$? \times 300 = 21\,000$$

Double *any* multiple of 5 up to 500

$$155 \times 2 = ? \quad 1015 \times 2 = ?$$

Halve any three-digit multiple of 10

$$240 \div 2 = ? \quad 760 \div 2 = ?$$

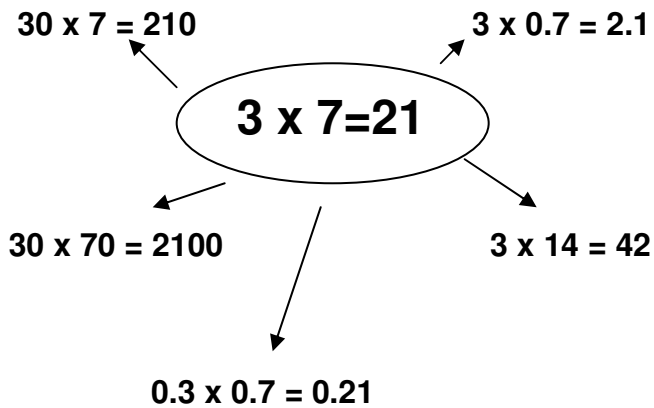
$$330 \times \frac{1}{2} = ? \quad ? \times \frac{1}{2} = 420$$

Multiply a two-digit whole number by *any* single-digit number, crossing the tens boundary

$$28 \times 3 \quad 18 \times 4$$

$$37 \times 9 = ? \quad 56 \times ? = 168$$

Fact webs can be very useful for helping children work out new facts from ones they already know:



Pencil and Paper Methods (\times and \div)

- Approximate first. Use *informal*/pencil and paper methods to support, record or explain multiplications and divisions.

Extend written methods to:

short multiplication of HTU or TU by U;

long multiplication of TU by TU;

short division of HTU by U (with whole number remainder).

Informal Written Methods

Multiplication

Grid Method (HTU \times U and TU \times TU)

437×6 is approximately $400 \times 7 = 2800$

x	400	30	7	
6	2400	180	42	= 2622

*Pupils who are finding it difficult to recall facts from $\times 6$ table can still access this calculation by partitioning the 6 into two easier numbers i.e. 4 and 2, or 5 and 1:

x	400	30	7	
2	800	60	14	= 874
4	1600	120	28	= 1748
				} = 2622

68×37 is approximately $70 \times 40 = 2800$

x	60	8	
30	1800	240	= 2040
7	420	56	= 476
			<u>2516</u>

Division

Using multiples of the divisor

$344 \div 6$ is between $300 \div 6 = 50$ and $360 \div 6 = 60$

$344 \div 6$	3 4 4
	<u>- 6 0</u> (10 x 6)
	2 8 4
	<u>- 1 2 0</u> (20 x 6)
	1 6 4
	<u>- 1 2 0</u> (20 x 6)
	4 4
	<u>- 4 2</u> (7 x 6)
	2

Answer: 57 remainder 2

Standard Written Methods

Multiplication

Partitioning

427×8 is approximately $400 \times 9 = 3600$

4 2 7	leading to	4 2 7
x 8	→	x 8
3 2 0 0 (400 x 8)		3 4 1 6
1 6 0 (20 x 8)		2 5
5 6 (7 x 8)		
<u>3 4 1 6</u>		

Long multiplication: TU x TU

63×46 is approximately $60 \times 50 = 3000$

5.7×7 is approximately $6 \times 7 = 42$

$$\begin{array}{r} 63 \\ \times 46 \\ \hline 2520 \text{ (63} \times 40\text{)} \\ 378 \text{ (63} \times 6\text{)} \\ \hline 2898 \end{array}$$

extend to simple decimals

$$\begin{array}{r} 5.0 \times 7 = 35.0 \\ 0.7 \times 7 = \underline{4.9} \\ 39.9 \end{array}$$

Division

Short division

$347 \div 7$ is approximately $350 \div 7 = 50$

$$\begin{array}{r} 7 \overline{) 347} \\ - 280 \quad 40 \times 7 \\ \hline 67 \\ - 63 \quad 9 \times 7 \\ \hline 4 \end{array} \quad \text{Answer: } 49 \text{ R } 4$$

Year Six

Mental Calculation Strategies (+ and -)

- Consolidate **all strategies from previous year**, including:

 - find a difference by counting up;
 - add or subtract the nearest Multiple of 10, 100 or 100 then adjust;
 - use the relationship between addition and subtraction;
 - add several numbers

Work out mentally by counting up from the smaller to the larger number:

$$7000 - 2867$$

$$3 + 30 + 100 + 4000 = 4133$$

Work out mentally:

$$317 + 289 = 606$$

double 300 plus 17 – 11

- Add/subtract 0.9, 1.9, 2.9, 1.1, etc. by adding or subtracting nearest whole number then adjusting:

a. $+ 7.8 = 12 - 0.1 = 11.9$

$$5.6 + 8.9 = 15 - 0.5 = 14.5$$

$$10.6 - 3.7 = 6.6 + 0.3 = 6.9$$

$$14.2 - 6.8 = 7.2 + 0.2 = 7.4$$

Work out mentally one fact such as $2.37 + 4.55$ or $8091 - 340$, and then state three related facts:-

$$2.37 + 4.55 = 6.92 \text{ therefore...}$$

$$6.92 - 4.55 = 2.37$$

$$6.92 - 2.37 = 4.55$$

$$4.55 + 2.37 = 6.92$$

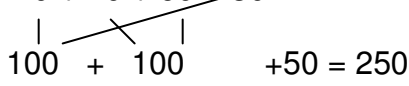
- Use $7705 - 2213 = 5492$ to work out:

10

$$7705 - 5494$$

$$7705 - 1213$$

Add mentally three or more multiples of

$$50 + 70 + 40 + 60 + 30$$


- Work mentally to complete questions like:

$$53 + ? + 38 = 115$$

$$77 + 64 + 96 = ?$$

Add sets of numbers such as:

$$64 + 67 + 68 + 62 \text{ recognising that this is equivalent to } (60 \times 4) + 21 = 261$$

- Use known number facts and place value to consolidate mental addition/subtraction

Add or subtract four-digit multiples of 100

$$6400 + 8900 \quad 5300 - 2600$$

$$4500 - 2100 = \hat{1}$$

$$3400 + 5600 = \hat{1}$$

- Find what to add to a decimal with units, 10ths and 100ths to make the next higher whole number or tenth

$$3.67 + 7.88 \quad 4 = \hat{1} + 7.9 = \hat{1}$$

Add or subtract a pair of decimal fractions each less than 1 and with up to two decimal places

$$0.07 + 0.34 \quad 0.8 + 0.73$$

$$0.89 + 1.29 = \hat{1}$$

$$0.67 - 0.09 = \hat{1}$$

$$0.56 - 0.17 = \hat{1}$$

Pencil and Paper Methods (+ and -)

- Use informal pencil and paper methods to support, record or explain additions and subtractions.

Extend written methods to column addition and subtraction of numbers involving decimals

Informal Written Methods

Addition – ThHTU + ThHTU, then numbers with any number of digits

Adding the most significant digits first

$\begin{array}{r} 5687 \\ + 1334 \\ \hline 6000 \\ 900 \\ 110 \\ 11 \\ \hline 7021 \end{array}$	$\begin{array}{r} 6885 \\ + 5538 \\ \hline 11000 \\ 1300 \\ 110 \\ 13 \\ \hline 12423 \end{array}$	extend to decimals	$\begin{array}{r} 128.17 \\ + 36.08 \\ \hline 100.00 \\ 50.00 \\ 14.00 \\ 0.10 \\ 0.15 \\ \hline 164.25 \end{array}$
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Compensation

$\begin{array}{r} 6775 \\ + 3688 \\ \hline 10775 \text{ (6775 + 4000)} \\ - 31 \\ \hline 10463 \end{array}$	$\begin{array}{r} 34.67 \\ + 27.88 \\ \hline 64.67 \text{ (34.67 + 30)} \\ - 2.12 \\ \hline 62.55 \end{array}$
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Subtraction – ThHTU – ThHTU, then with any number of digits

Counting up (complementary addition)

$$\begin{array}{r} 7834 \\ - 4657 \\ \hline 43 \text{ (4700)} \\ 300 \text{ (5000)} \\ \hline 2834 \text{ (7834)} \\ 3177 \end{array}$$

Compensation

$$\begin{array}{r} 7834 \\ - 4657 \\ \hline 3134 \text{ (7834 - 4700)} \\ + 43 \\ \hline 3177 \end{array}$$

Standard Written Methods

Addition

Using 'carrying'

$$\begin{array}{r} 5687 \\ + 1334 \\ \hline 7021 \\ 111 \end{array}$$

$$\begin{array}{r} 6885 \\ + 5538 \\ \hline 12423 \\ 111 \end{array}$$

$$\begin{array}{r} 45735 \\ + 1335 \\ \hline 1264 \\ 427 \\ \hline 48761 \\ 112 \end{array}$$

Extend to decimals:-

$$128.17 + 36.08$$

$$\begin{array}{r} 128.17 \\ + 36.08 \\ \hline 164.25 \\ 11 \end{array}$$

Subtraction

Decomposition

$$\begin{array}{r} 513161 \\ 6475 \\ - 2586 \\ \hline 3889 \end{array}$$

$$\begin{array}{r} 314101 \\ 345.14 \\ - 8.27 \\ \hline 336.87 \end{array}$$

Mental Calculation Strategies (× and ÷)

- Use related facts and doubling or halving. For example:
 - double or halve the most significant digit first;
 - to multiply by 25, multiply by 100, then divide by 4;
 - double one number and halve the other;
 - find the x24 table by doubling the x6 table twice

Use related facts and doubling/halving
Double $237 = 400 + 60 + 14 = 474$
Half of $876 = 400 + 35 + 3 = 478$

Double a number ending in 5 and halve the other number:
 16×15
 $8 \times 30 = 240$

Halve/double one number in a calculation, then double/halve the answer:
 $16 \times 15 = (8 \times 15) \times 2$
 $= 120 \times 2$
 $= 240$

To multiply by 15, multiply by 10, halve the answer, then add the two parts:
 $43 \times 15 = 430 + 215$
 $= 645$

To multiply by 25, multiply by 100, then divide by 4:
 $34 \times 25 = 3400 \div 4$
 $= 850$

Work out x24 table facts by doubling the x6 table twice:
 $6 \times 24 = 36 \times 2 \times 2$
 $= 72 \times 2$
 $= 144$

Use combinations of facts to work out
Other multiples:

$1 \times 43 = 43$
 $2 \times 43 = 86$
 $4 \times 43 = 172$
 $8 \times 43 = 344$
 $16 \times 43 = 688$



$33 \times 43 = (688 \times 2) + 43$
 $= 1376 + 43$
 $= 1419$

Explain how to find sixths and twelfths by halving thirds, or twentieths by halving tenths:
one sixth of 93 is one half of 31
 $= 15.5$
one twelfth of 93 is therefore 7.75

Use factors, for example:
 $43 \times 16 = 43 \times 4 \times 2 \times 2$
 $= 172 \times 2 \times 2$
 $= 344 \times 2$
 $= 688$

- Use closely related facts:
for example, multiply by 49 or 51 by multiplying by 50 then adjusting.

$23 \times 51 = 1150 + 23$
 $= 1173$
 $17 \times 49 = 850 - 17$
 $= 833$

Develop the x17 table by adding facts from the x10 and x7 tables

$4 \times 17 = (4 \times 10) + (4 \times 7)$
 $= 40 + 28$
 $= 68$

□ Partition

$$\begin{aligned} 96 \times 7 &= 630 + 42 \\ &= 672 \\ 84 \times 14 &= 840 + 336 \\ &= 1176 \\ 6.7 \times 6 &= (6 \times 6) + (0.7 \times 6) \\ &= 36 + 4.2 \text{ leading to...} \\ 6.7 \times 6 &= 36 + 4.2 \\ &= 40.2 \end{aligned}$$

□ Use the relationship between multiplication and division

Continue to recognise that knowing one calculation means you know another three:
 $6.7 \times 6 = 40.2$ therefore....

$$\begin{aligned} 6 \times 6.7 &= 40.2 \\ 40.2 \div 6 &= 6.7 \\ 40.2 \div 6.7 &= 6 \end{aligned}$$

Recognise that if:
 $4 \times 80 = 320$ then $\frac{1}{4}$ of 320 = 80 and
 $\frac{1}{8}$ of 320 = 40

Answer oral questions such as:
 If $1.6 \times 1.2 = 1.92$,
 What is $1.92 \div 1.6$?

Pencil and Paper Methods (\times and \div)

- Approximate first. Use informal pencil and paper methods to support, record or explain multiplications and divisions.

Extend written methods to:

multiplication of ThHTU \times U (short multiplication);

short multiplication of numbers involving decimals;

long multiplication of a three-digit by a two-digit whole number;

short division of TU or HTU by U (mixed number answer);

division of HTU by TU (long division, whole number answer);

short division of numbers involving decimals.

Informal Written Methods

Multiplication

Approximate first

Grid method (ThHTU \times U and HTU \times TU)

3578×7 is approximately $3500 \times 7 = 24500$

x	3000	500	70	8	
7	21000	3500	490	56	= 25046

456×37 is approximately $450 \times 40 = 18000$

x	400	50	6	
30	12000	1500	180	13680
7	2800	350	42	+ $\frac{3192}{16872}$

6.93×6 is approximately $7 \times 6 = 42$

x	6	0.9	0.03	
6	36	5.4	0.18	= 41.58

Division

Approximate first.

Using multiples of the divisor - HTU \div TU

$867 \div 24$ is approximately $900 \div 25 = 36$

$867 \div 24$

	leading to	
$\begin{array}{r} 867 \\ - 240 \quad (10 \times 24) \\ \hline 627 \\ - 240 \quad (10 \times 24) \\ \hline 387 \\ - 240 \quad (10 \times 24) \\ \hline 147 \\ - 144 \quad (6 \times 24) \\ \hline 3 \end{array}$	\rightarrow	$\begin{array}{r} 867 \\ - 720 \quad (30 \times 24) \\ \hline 147 \\ - 144 \quad (6 \times 24) \\ \hline 3 \end{array}$

Answer: $36 \frac{3}{24} \rightarrow 36\frac{1}{8}$

Standard Written Methods

Multiplication

Partitioning

Short multiplication: ThHTU \times U

3578×7 is approximately $3500 \times 7 = 24500$

	leading to	
$\begin{array}{r} 3578 \\ \times 7 \\ \hline 21000 \quad (3000 \times 7) \\ 3500 \quad (500 \times 7) \\ 490 \quad (70 \times 7) \\ 56 \quad (8 \times 7) \\ \hline 25046 \end{array}$	\rightarrow	$\begin{array}{r} 3578 \\ \times 7 \\ \hline 25046 \\ 455 \end{array}$

Long multiplication: HTU \times TU

473×26 is approximately $450 \times 30 = 13500$

$$\begin{array}{r} 473 \\ \times 26 \\ \hline 9460 \quad (473 \times 20) \\ 2400 \quad (400 \times 6) \\ 420 \quad (70 \times 6) \\ 18 \quad (3 \times 6) \\ \hline 12298 \end{array}$$

1

518×42 is approximately $500 \times 40 = 20000$

$$\begin{array}{r} 518 \\ \times 42 \\ \hline 20720 \quad (518 \times 40) \\ 1036 \quad (518 \times 2) \\ \hline 21756 \end{array}$$

Extend to decimals with up to two decimal places

6.87 x 6 is approximately 7 x 6 = 42

$$\begin{array}{r} 6.87 \times 6 \\ 6.00 \times 6 = 36.00 \\ 0.80 \times 6 = 4.80 \\ 0.07 \times 6 = 0.42 \\ \hline 41.22 \end{array}$$

5.34 x 48 is approximately 5.34 x 50 = 267

$$\begin{array}{r} 5.34 \times 48 \\ 5.00 \times 40 = 200.00 \\ 5.00 \times 8 = 40.00 \\ 0.30 \times 40 = 12.00 \\ 0.30 \times 8 = 2.40 \\ 0.04 \times 40 = 1.60 \\ 0.04 \times 8 = 0.32 \\ \hline 256.32 \end{array}$$

Division

Long division HTU ÷ TU

896 ÷ 24 is approximately 900 ÷ 25 = 36

$$\begin{array}{r} 24 \overline{) 896} \\ \underline{- 720} \quad (30 \times 24) \\ 176 \\ \underline{- 168} \quad (7 \times 24) \\ 8 \end{array}$$

Answer: **37** $\frac{8}{24}$ → **37 $\frac{1}{3}$**

$$\begin{array}{r} 24 \overline{) 896} \\ \underline{- 720} \quad (30 \times 24) \\ 176 \\ \underline{- 168} \quad (7 \times 24) \\ 8.0 \\ \underline{- 7.2} \quad (0.3 \times 24) \\ 0.80 \\ \underline{- 0.72} \quad (0.03 \times 24) \end{array}$$

Answer: **37.33**

Fact Webs can help children derive new number facts from ones they already know and are particularly useful as preparation for more complex multiplication and division. Starting with a simple fact from any times table, children use doubling/halving, multiplying by 10, 100 etc., to generate new facts.

