"Together we grow"

Emmaville Primary School

Calculation Policy

A whole School Approach to Mental and Written Calculations

A Guide for Teachers and Parents

Aims of the policy.

This policy is designed to create a common way of teaching calculation strategies at Emmaville Primary School, and to provide detailed guidance and information to staff and parents to enable them to effectively support the development of children's calculation skills. It has been devised to meet the requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is designed to give pupils a consistent and smooth progression of learning in calculations across the school. Please note that the early learning in number and calculation in Reception follows the 'Development Matters' Early Years Foundation Stage document, and this calculation policy is designed to build on progressively from the content and methods established in the EYFS. The consistent use of CPA (Concrete, Pictorial, Abstract) approach helps children develop mastery in both written and mental methods across all the operations in an efficient, reliable way and develops children's confidence in their understanding of methods we use.

Reception

Mathematics involves providing children with opportunities to develop and improve their skills in counting, understanding and using numbers, calculating simple addition and subtraction problems; and to describe shapes, spaces, and measures.

Year 1 & 2

Children develop the core ideas that underpin all calculation. They begin by connecting calculation with counting on and counting back, but they also learn that understanding wholes and parts will enable them to calculate efficiently and accurately, and with greater flexibility. They learn how to use an understanding of 10s and 1s to develop their calculation strategies, especially in addition and subtraction.

Year 3 & 4

Children develop the basis of written methods by building their skills alongside a deep understanding of place value. They should use known addition/subtraction and multiplication/division facts to calculate efficiently and accurately, rather than relying on counting. Children use place value equipment to support their understanding, but not as a substitute for thinking.

Year 5 & 6

Children build on secure foundations in calculation, and develop fluency, accuracy and flexibility in their approach to the four operations. They work with whole numbers and adapt their skills to work with decimals, and they continue to develop their ability to select appropriate, accurate and efficient operations.
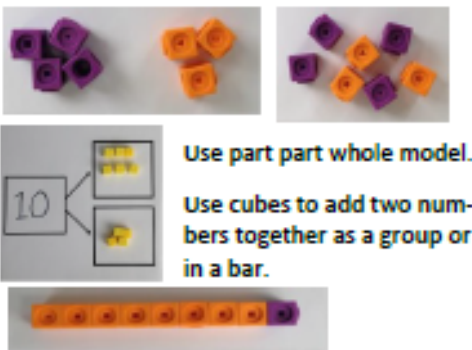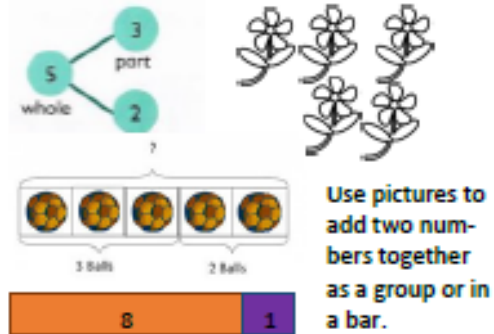

Key Language:
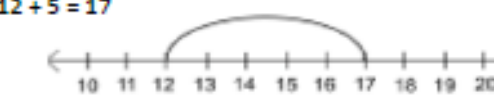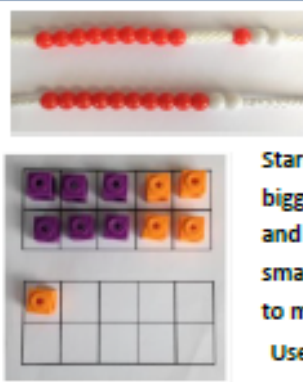
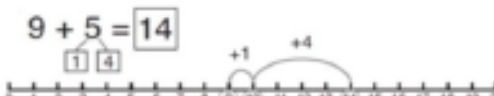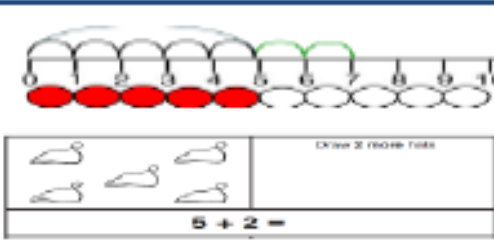- Addition: sum, total, parts and wholes, add, altogether, more, 'is equal to', 'is the same as'
- Subtraction: take away, less than, the difference, subtraction, minus, fewer, decrease
- Multiplication: double, times, multiplied by, the product of, lots of, equal groups
- Division: share, group, divide, divided by, half.

Overview of progression in calculation

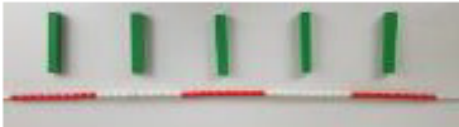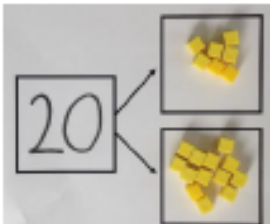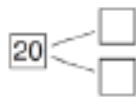|	EYFS / Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on – using cubes. Regrouping to make 10 using ten frame.	Adding three single digits. Use of base 10 to combine two numbers.	Column method- regrouping. Using place value counters (up to 3 digits).	Column method – regrouping. (up to 4 digits)	Column method – regrouping. Use of place value counters for adding decimals.	Column method – regrouping. Abstract methods. Place value counters to be used for adding decimal numbers.
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10 using the ten frame	Counting back Find the difference Part whole model Make 10 Use of base 10	Column method with regrouping. (up to 3 digits using place value counters)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. Abstract for whole numbers. Start with place value counters for decimals – with the same amount of decimal places.	Column method with regrouping. Abstract methods. Place value counters for decimals – with different amounts of decimal places.
Multiplication	Recognising and making equal groups. Doubling Counting in multiples. Use cubes, Numicon and other objects in the classroom.	Arrays showing commutative multiplication.	Arrays 2digit x 1digit using base 10	Column multiplication introduced with place value counters. (2 and 3 digit multiplied by 1 digit)	Column multiplication Abstract only but might need a repeat of year 4 first (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication Abstract methods (multi-digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups? Use cubes and draw round 3 cubes at a time.	Division as grouping Division with arrays linking to multiplication Repeated subtraction	Division with remainder – using lollipop sticks, times tables facts and repeated subtraction. 2 digit divided by 1 digit using base 10 or place value counters	Division with a remainder Short division (up to 3 digits by 1 digit – concrete and pictorial)	Short division (up to 4 digits by a 1 digit number including remainders)	Short division Long division with place value counters (up to 4 digits by a 2 digit number) Children should exchange into the tenths and hundredths column too
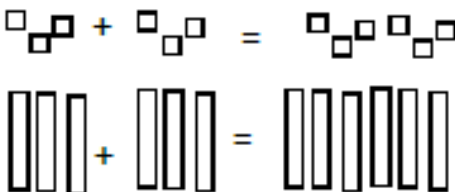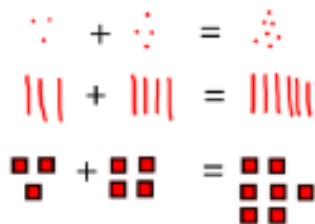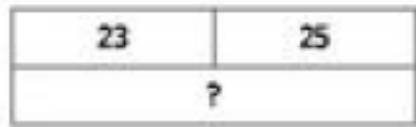
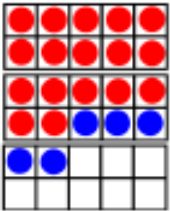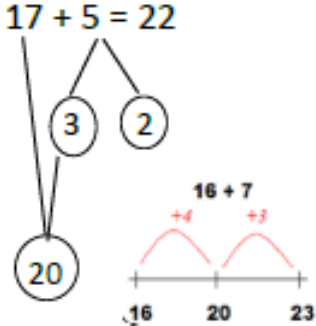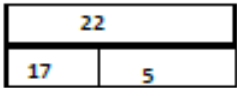| Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	Use part part whole model. Use cubes to add two numbers together as a group or in a bar.	Use pictures to add two numbers together as a group or in a bar. 3 Balls 2 Balls 8 | 1	$4 + 3 = 7$ $10 = 6 + 4$ Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	$12 + 5 = 17$ Start at the larger number on the number line and count on in ones or in one jump to find the answer.	$5 + 12 = 17$ Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. *This is an essential skill for column addition later.*	$6 + 5 = 11$ Start with the bigger number and use the smaller number to make 10. Use ten frames.	 $3 + 9 =$ Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10. $9 + 5 = \boxed{14}$ $\boxed{1}\boxed{4}$	$7 + 4 = 11$ If I am at seven, how many more do I need to make 10. How many more do I add on now?
Represent & use number bonds and related subtraction facts within 20	 2 more than 5.	 $5 + 2 =$	Emphasis should be on the language '1 more than 5 is equal to 6.' '2 more than 5 is 7.' '8 is 3 more than 5.'

Y1

ADDITION +
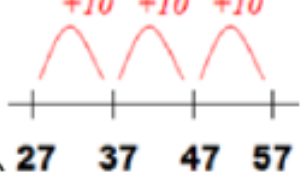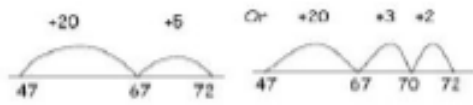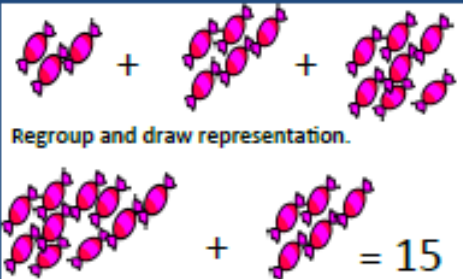
Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	50= 30 = 20 Model using dienes and bead strings	3 tens + 5 tens = _____ tens 30 + 50 = _____ Use representations for base ten.	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$
Use known number facts Part part whole	20 Children explore ways of making numbers within 20	20 $\square + \square = 20$ $\quad 20 - \square = \square$ $\square + \square = 20$ $\quad 20 - \square = \square$	$\square + 1 = 16$ $\quad 16 - 1 = \square$ $1 + \square = 16$ $\quad 16 - \square = 1$
Using known facts	$\square + \square = \square$ $\square + \square = \square$	$\cdot + \cdot = \cdot$ $||| + |||| = ||||||$ Children draw representations of H,T and O	$3 + 4 = 7$ leads to $30 + 40 = 70$ leads to $300 + 400 = 700$
Bar model	$3 + 4 = 7$	$7 + 3 = 10$		23	25	 	?	 $23 + 25 = 48$

Objective & Strategy	Concrete	Pictorial	Abstract
Add a two digit number and ones	$17 + 5 = 22$ Use ten frame to make 'magic ten Children explore the pattern. $17 + 5 = 22$ $27 + 5 = 32$	Use part part whole and number line to model. $17 + 5 = 22$ 3 2 20 $16 + 7$ $+4$ $+3$ 16 20 23	$17 + 5 = 22$ Explore related facts $17 + 5 = 22$ $5 + 17 = 22$ $22 - 17 = 5$ $22 - 5 = 17$ 22 / 17 / 5
Add a 2 digit number and tens	$25 + 10 = 35$ Explore that the ones digit does not change	$27 + 30$ $+10$ $+10$ $+10$ 27 37 47 57	$27 + 10 = 37$ $27 + 20 = 47$ $27 + \square = 57$
Add two 2-digit numbers	Model using dienes , place value counters and numicon	$+20$ $+5$ Or $+20$ $+3$ $+2$ 47 67 72 47 67 70 72 Use number line and bridge ten using part whole if necessary.	$25 + 47$ $20 + 5$ $40 + 7$ $20 + 40 = 60$ $5 + 7 = 12$ $60 + 12 = 72$
Add three 1-digit numbers	Combine to make 10 first if possible, or bridge 1o then add third digit	+ + Regroup and draw representation. + $= 15$	$4 + 7 + 6 = \boxed{10} + \boxed{7}$ 10 $= \boxed{17}$ Combine the two numbers that make/bridge ten then add on the third.
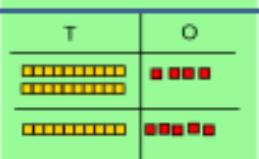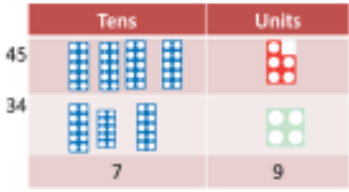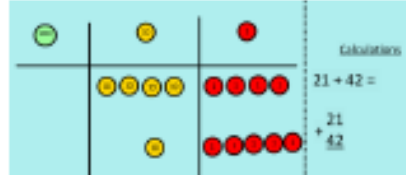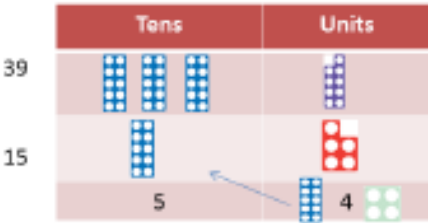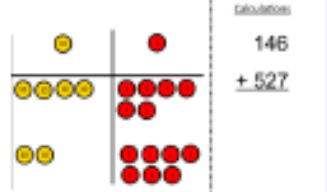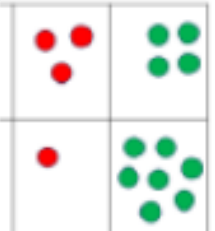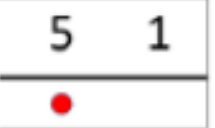
Y2 ADDITION +
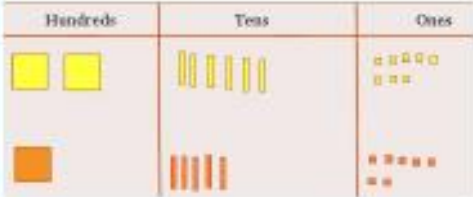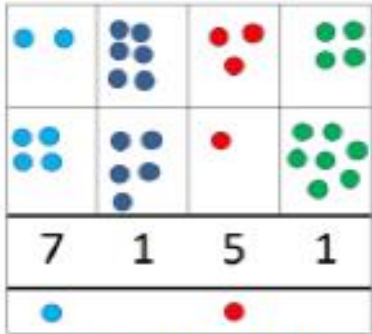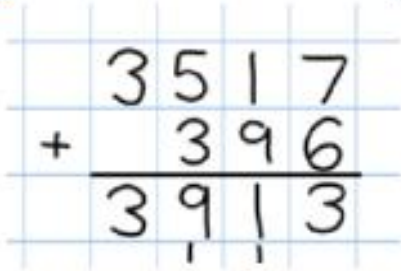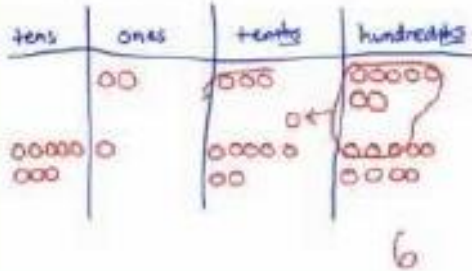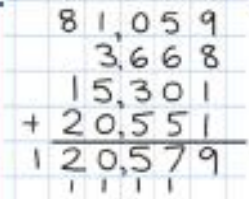
Objective & Strategy	Concrete	Pictorial	Abstract

Column Addition—no regrouping (friendly numbers)

Add two or three 2 or 3-digit numbers.

T	O

Model using Dienes or nu-micon

Add together the ones first, then the tens.

Tens	Units
45	
34	
7	9

$21 + 42 =$

21
$+ 42$

Move to using place value counters

Pictorial: Children move to drawing the counters using a tens and one frame.

tens	ones

Abstract:

$$\begin{array}{r} 2\ 2\ 3 \\ +\ 1\ 1\ 4 \\ \hline 3\ 3\ 7 \end{array}$$

Add the ones first, then the tens, then the hundreds.

Column Addition with regrouping.

Tens	Units
39	
15	
5	4

Exchange ten ones for a ten. Model using numicon and pv counters.

146
$+ 527$

Pictorial:

5	1

Children can draw a representation of the grid to further support their understanding, carrying the ten underneath the line

Abstract:

$$\begin{array}{rcl} 20 & + & 5 \\ 40 & + & 8 \\ \hline 60 & + & 13 & = 73 \end{array}$$

Start by partitioning the numbers before formal column to show the exchange.

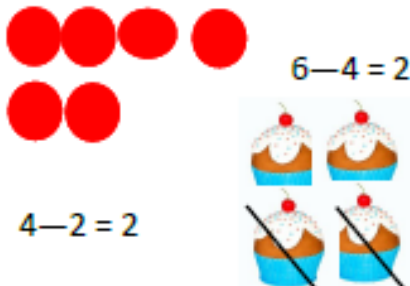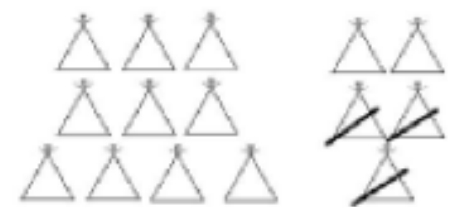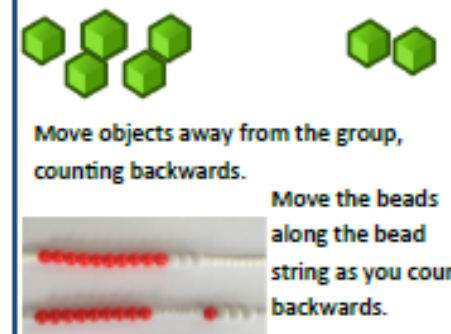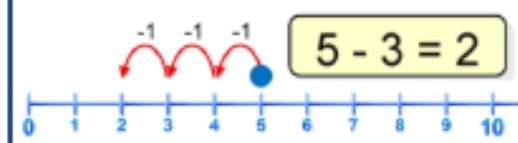$$\begin{array}{r} 536 \\ +\ 85 \\ \hline 621 \\ 1\ 1 \end{array}$$

Y3 ADDITION +
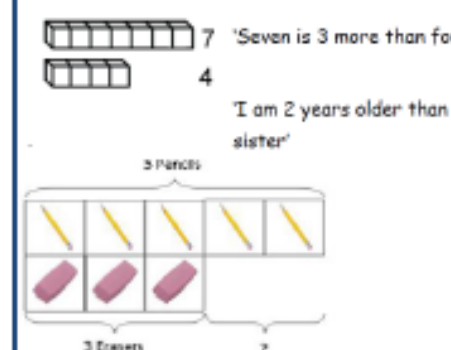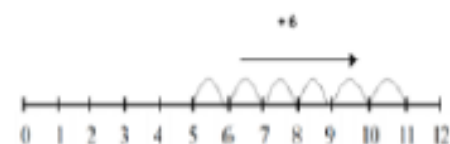
Objective & Strategy	Concrete	Pictorial	Abstract	Y4-6
Y4—add numbers with up to 4 digits	Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.	Draw representations using pv grid.	$$\begin{array}{r} 3517 \\ +\ 396 \\ \hline 3913 \end{array}$$ Continue from previous work to carry hundreds as well as tens. Relate to money and measures.	ADDITION +
Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As year 4 Introduce decimal place value counters and model exchange for addition.	2.37 + 81.79	$$\begin{array}{r} 72.8 \\ +\ 54.6 \\ \hline 127.4 \\ 1\ 1 \end{array}$$ $$\begin{array}{r} £23 . 59 \\ +\ £7 . 55 \\ \hline £31 . 14 \end{array}$$	
Y6—add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5	$$\begin{array}{r} 81,059 \\ 3,668 \\ 15,301 \\ +\ 20,551 \\ \hline 120,579 \\ 1\ 1\ 1\ 1 \end{array}$$ Insert zeros for place holders. $$\begin{array}{r} 23 . 361 \\ 9 . 080 \\ 59 . 770 \\ +\ 1 . 300 \\ \hline 93 . 511 \end{array}$$	
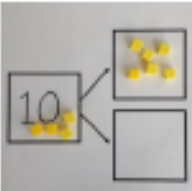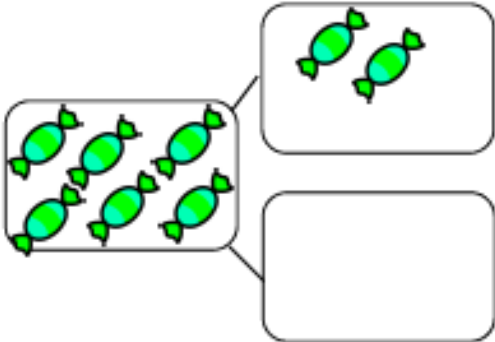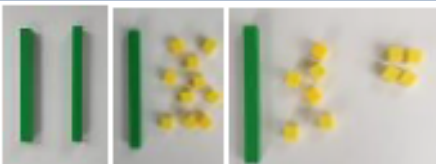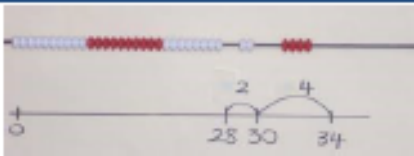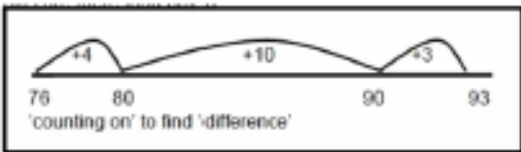
Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters , cubes etc to show how objects can be taken away. $6—4 = 2$ $4—2 = 2$	 $15 - 3 = \boxed{12}$ Cross out drawn objects to show what has been taken away.	$7—4 = 3$ $16—9 = 7$
Counting back	Move objects away from the group, counting backwards. Move the beads along the bead string as you count backwards.	$-1 \quad -1 \quad -1$ $\boxed{5 - 3 = 2}$ 0 1 2 3 4 5 6 7 8 9 10 Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?
Find the Difference	Compare objects and amounts 7 'Seven is 3 more than four' 4 'I am 2 years older than my sister' 3 Pencils 3 Erasers 7 Lay objects to represent bar model.	Count on using a number line to find the difference. +6 0 1 2 3 4 5 6 7 8 9 10 11 12	Hannah has12 sweets and her sister has 5. How many more does Hannah have than her sister.?

Y1 SUBTRACTION -
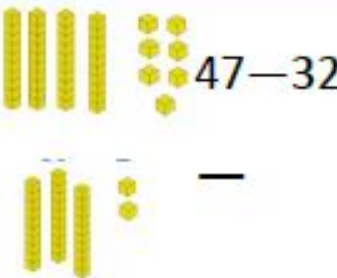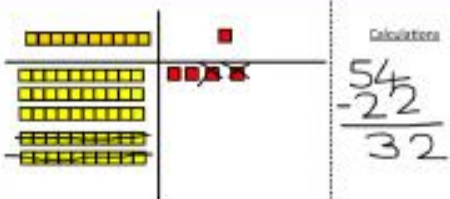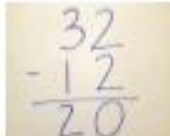
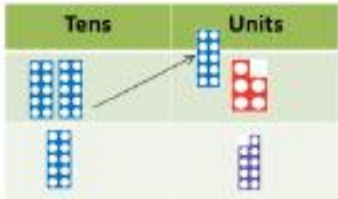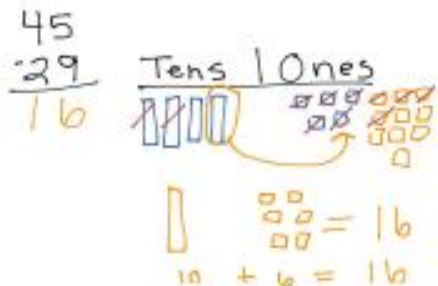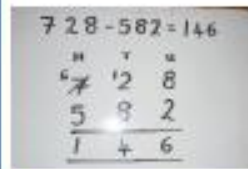| Objective & Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the arts, what s the other part? 10—6 = 4	Use pictorial representations to show the part.	Move to using numbers within the part whole model. 12 5 7
Make 10	14—9 Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	13—7 $13 - 7 = 6$ Jump back 3 first, then another 4. Use ten as the stopping point.	16—8 How many do we take off first to get to 10? How many left to take off?
Bar model	5—2 = 3		8 2 10 = 8 + 2 10 = 2 + 8 10—2 = 8 10—8 = 2

Y1 SUBTRACTION -

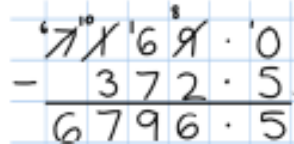| Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'	20 — 4 =	$20 - 4 = 16$
Partitioning to subtract without regrouping. *'Friendly numbers'*	$34 - 13 = 21$ Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off. $43 - 21 = 22$	$43 - 21 = 22$
Make ten strategy *Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.*	34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	+4 +10 +3 76 80 90 93 'counting on' to find 'difference' Use a number line to count on to next ten and then the rest.	$93 - 76 = 17$

Y2

SUBTRACTION
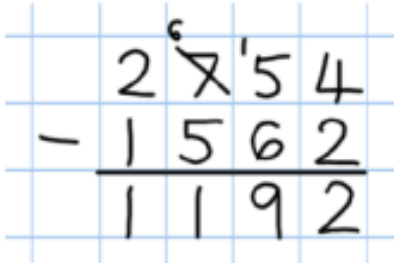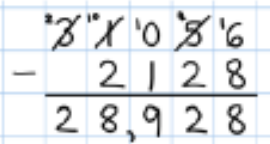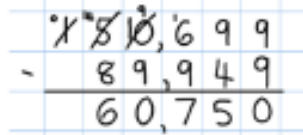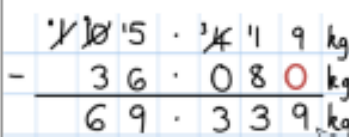
Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	$47-32$ Use base 10 or Numicon to model	Darw representations to support understanding	$47-24=23$ $$-\frac{40+7}{20+4}$$ $$\overline{20+3}$$ Intermediate step may be needed to lead to clear subtraction understanding. $$32$$ $$-12$$ $$\overline{20}$$
Column subtraction with regrouping	Tens Units Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into tten ones. Use the phrase 'take and make' for exchange.	$$45$$ $$-29$$ $$\overline{16}$$ Tens | Ones $$10+6=16$$ Children may draw base ten or PV counters and cross off.	$836-254=582$ Begin by partitioning into pv columns $728-582=146$ Then move to formal method.

Y3 SUBTRACTION

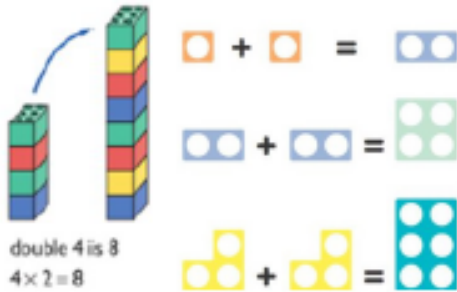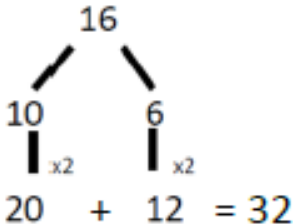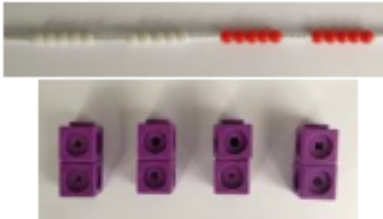| Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. *Introduce decimal subtraction through context of money*	234 - 179 Model process of exchange using Numicon, base ten and then move to PV counters.	Children to draw pv counters and show their exchange—see Y3	 Use the phrase 'take and make' for exchange
Year 5- Subtract with at least 4 digits, including money and measures. *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal*	As Year 4	Children to draw pv counters and show their exchange—see Y3	 Use zeros for placeholders.
Year 6—Subtract with increasingly large and more complex numbers and decimal values.			
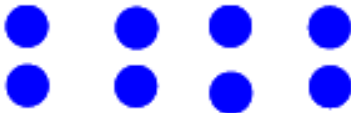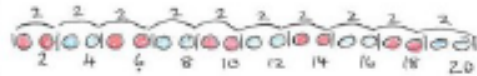
SUBTRACTION
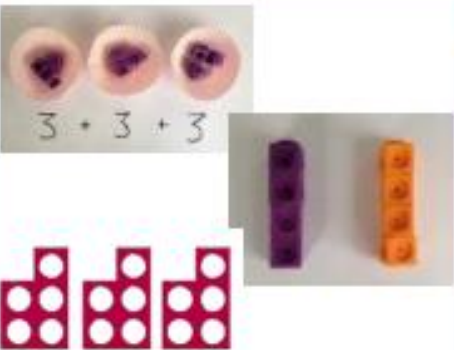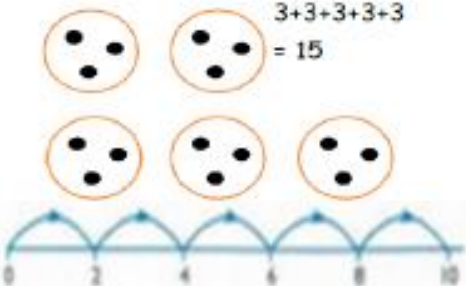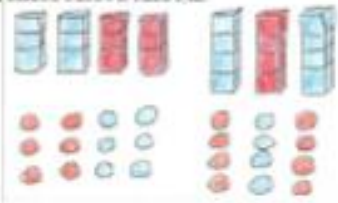
Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipultives including cubes and Numicon to demonstrate doubling □ + □ = □□ □□ + □□ = 🞖🞖 🞖🞖 + 🞖🞖 = 🞖🞖🞖 double 4 is 8 $4 \times 2 = 8$	Draw pictures to show how to double numbers Double 4 is 8	Partition a number and then double each part before recombining it back together. 16 10 6 x2 x2 $20 + 12 = 32$
Counting in multiples	Count the groups as children are skip counting, children may use their fingers as they are skip counting.	 ● ● ● ● ● ● ● ● Children make representations to show counting in multiples. 2 2 2 2 2 2 2 2 2 2 4 6 8 10 12 14 16 18 20	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Making equal groups and counting the total	Use manipulatives to create equal groups. □ x □ = 8	Draw 🐟 to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8
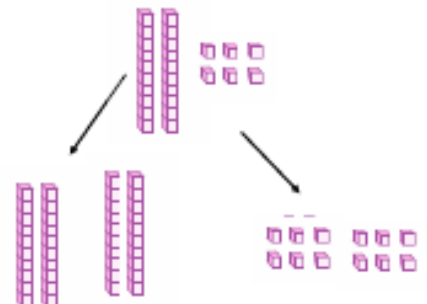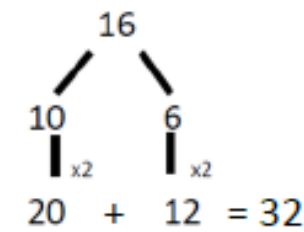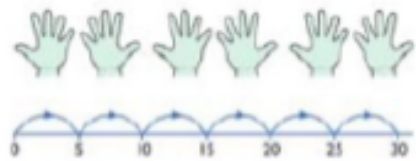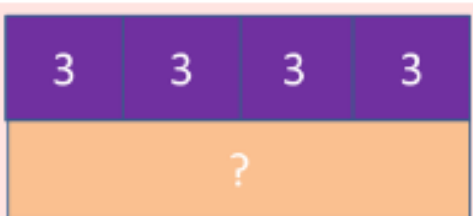
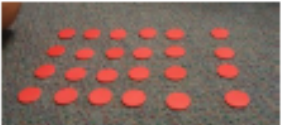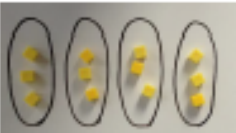| Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	3 + 3 + 3 Use different objects to add equal groups	Use pictorial including number lines to solve prob There are 3 sweets in one bag. How many sweets are in 5 bags altogether? 3+3+3+3+3 = 15 	Write addition sentences to describe objects and pictures. 2 + 2 + 2 + 2 + 2 = 10
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding. 	$3 \times 2 = 6$ $2 \times 5 = 10$
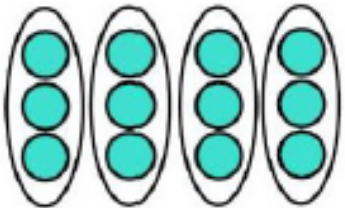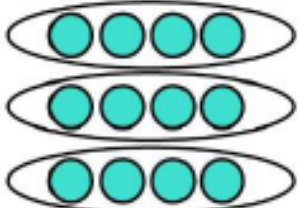
MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters. $$40 + 12 = 52$$	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together. 16, 10, 6, x2, x2, $$20 + 12 = 32$$
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models. $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$?	Number lines, counting sticks and bar models should be used to show representation of counting in multiples. 3 3 3 3 ?	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $$4 \times 3 = \square$$

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative	Create arrays using counters and cubes and Numicon. Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.	Use representations of arrays to show different calculations and explore commutativity.	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition. $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$
Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*			$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$ Show all 8 related fact family sentences.

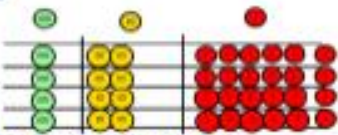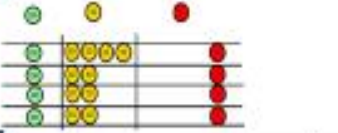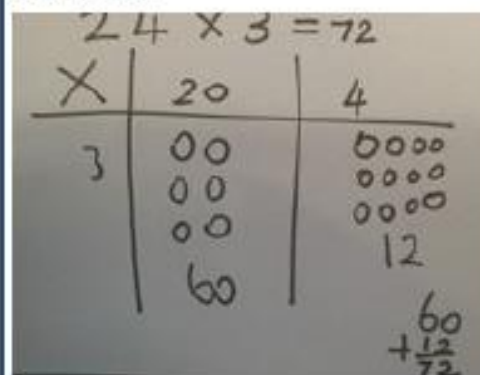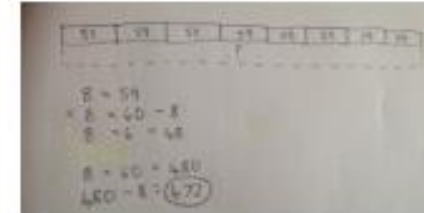**Y2**
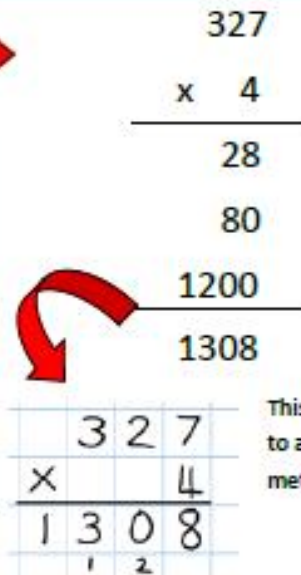
MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Grid method	Show the links with arrays to first introduce the grid method. 4 rows of 10 4 rows of 3 Move onto base ten to move towards a more compact method. 4 rows of 13 Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows Calculations 4 x 126 Fill each row with 126 Calculations 4 x 126 Add up each column, starting with the ones making any exchanges needed Then you have your answer.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below. Bar model are used to explore missing numbers $4 \times \boxed{} = 20$ 	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | ×	30	5
---	---	---
7	210	35	 $210 + 35 = 245$ Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

Objective & Strategy	Concrete	Pictorial	Abstract
Grid method recap from year 3 for 2 digits x 1 digit Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows Calculations 4 x 126 Fill each row with 126 Add up each colu... ...es making any exchanges needed	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below. 24 × 3 = 72	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. 	×	30	5
---	---	---
7	210	35	 210 + 35 = 245
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642 Hundreds Tens Ones It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside		×	300	20	7
4	1200	80	28	 The grid method my be used to show how this relates to a formal written method. Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.	327 x 4 _____ 28 80 1200 1308 327 × 4 _____ 1308 This may lead to a compact method.

Y4

MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract

Column Multiplication for 3 and 4 digits x 1 digit.

It is important at this stage that they always multiply the ones first.

Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping.   321 x 2 = 642

×	300	20	7
4	1200	80	28

327

x   4

28

80

1200

1308

This will lead to a compact method.

Column multiplication

Manipulatives may still be used with the corresponding long multiplication modelled alongside.

Continue to use bar modelling to support problem solving

18 x 3 on the first row

(8 x 3 =24, carrying the 2 for 20, then 1 x 3)

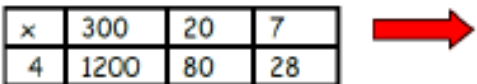18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first
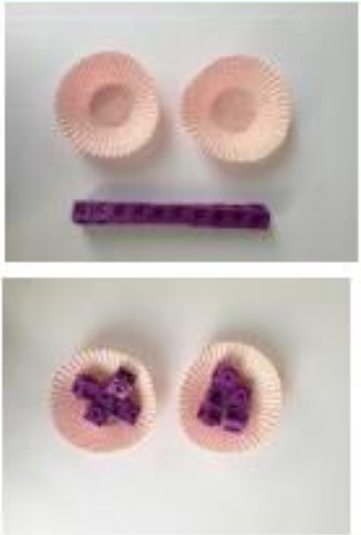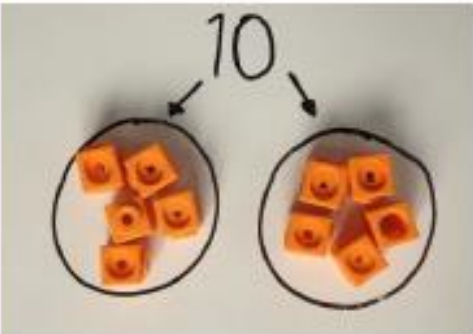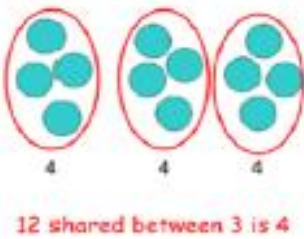
Y5-6

MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. $$\begin{array}{r} 3 \cdot 1\ 9 \\ \times\ 8 \\ \hline 2\ 5 \cdot 5\ 2 \end{array}$$
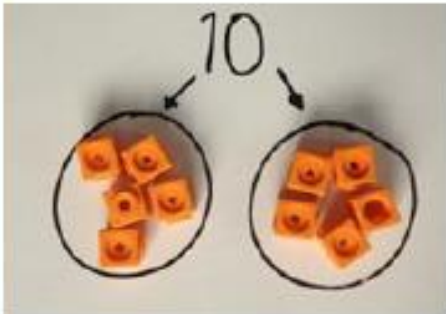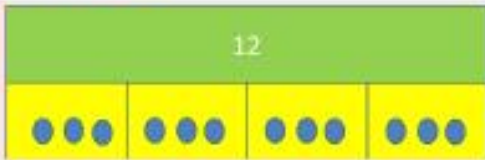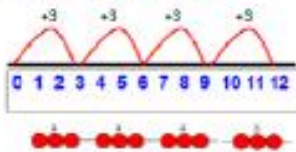
Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing *Use Gordon ITPs for modelling*	 *I have 10 cubes, can you share them equally in 2 groups?*	Children use pictures or shapes to share quantities. 8 shared between 2 is 4 Sharing: 4 4 4 12 shared between 3 is 4	12 shared between 3 is 4

Y1
DIVISION ÷
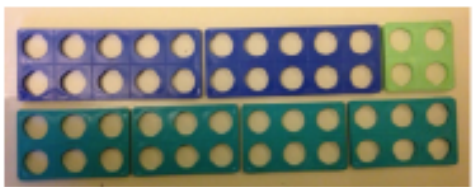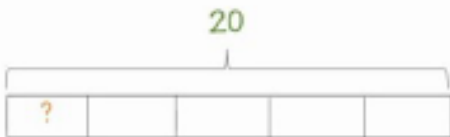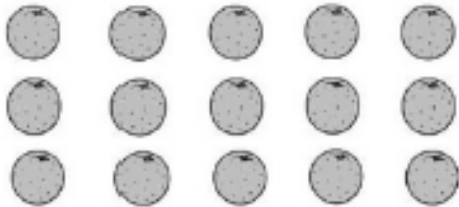
Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing

I have 10 cubes, can you share them equally in 2 groups? | Children use pictures or shapes to share quantities.

$8 \div 2 = 4$

Children use bar modelling to show and support understanding.

$12 \div 4 = 3$ | $12 \div 3 = 4$ |
| Division as grouping | Divide quantities into equal groups.

Use cubes, counters, objects or place value counters to aid understanding.

 | Use number lines for grouping

$12 \div 3 = 4$

Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.

$20 \div 5 = ?$
$5 \times ? = 20$ | $28 \div 7 = 4$

Divide 28 into 7 groups. How many are in each group? |

Y2
DIVISION ÷

Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding. 24 divided into groups of 6 = 4 $$96 \div 3 = 32$$ 	Continue to use bar modelling to aid solving division problems. $$20 \div 5 = ?$$ $$5 \times ? = 20$$	How many groups of 6 in 24? $$24 \div 6 = 4$$
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$

Y3
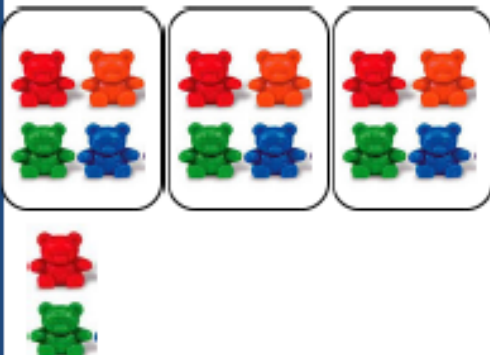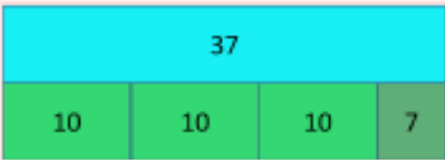
DIVISION ÷
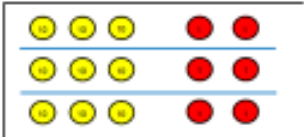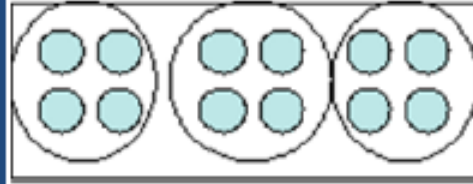
Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders.	$14 \div 3 =$ Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. Draw dots and group them to divide an amount and clearly show a remainder. remainder 2 Use bar models to show division with remainders. 	Complete written divisions and show the remainder using r. $29 \div 8 = 3$ REMAINDER 5 dividend divisor quotient remainder

Example without remainder:
40 ÷ 5
Ask "How many 5s in 40?"

$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5$ = 8 fives

0   5   10   15   20   25   30   35   40

Example with remainder:
38 ÷ 6

$6 + 6 + 6 + 6 + 6 + 6 + 2$ = 6 sixes with a remainder of 2

0   6   12   18   24   30   36   38

For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.

Y3

DIVISION ÷

Objective & Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division	96 ÷ 3 Tens Units 3 2 Use place value counters to divide using the bus stop method alongside Calculations 42 ÷ 3 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over. We exchange this ten for ten ones and then share the ones equally among the groups. We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups. Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. $$\begin{array}{r} 2\ \ 1\ \ 8 \\ \hline 4\,|\,8\ \ 7\ \ 2 \end{array}$$ Move onto divisions with a remainder. $$\begin{array}{r} 8\ \ 6\ \ r\ 2 \\ \hline 5\,|\,4\ \ 3\ \ 2 \end{array}$$ Finally move into decimal places to divide the total accurately. $$\begin{array}{r} 1\ \ 4\ \ .\ \ 6 \\ \hline 3\ \ 5\,|\,5\ \ 1\ \ 1\ \ .\ \ 0 \end{array}$$ $$\begin{array}{r} 0\ 6\ 6\ 3\ r\ 5 \\ \hline 8)5^5 3^5 0^2 9 \end{array}$$

Long Division

Step 1—a remainder in the ones

```
        h  t  o
      0  4  1 R1
  4 ) 1  6  5
```

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

```
        th h  t  o
      0  4  0  0 R7
  8 ) 3  2  0  7
```

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

Long Division

```
         h  t  o
         0  6  1
      4 ) 2  4  7
            – 4
               3
```

When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

```
      th  h  t  o
       0  4  0  2
    4 ) 1  6  0  9
               – 8
                  1
```

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1,609$

Long Division

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o **2** 2)**5** 8	t o 2 2)5 8 **- 4** **1**	t o 2 9 2)5 **8** - 4 ↓ 1 **8**
Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 **9** **2**)5 8 - 4 **1 8**	t o 2 9 2)5 8 - 4 **1 8** **- 1 8**	t o **2 9** 2)5 8 - 4 1 8 - 1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.

Long Division

Step 2—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$$\begin{array}{r} h\ t\ o \\ 1 \\ 2\,\overline{)2\ 7\ 8} \end{array}$$	$$\begin{array}{r} h\ t\ o \\ 1 \\ 2\,\overline{)2\ 7\ 8} \\ -2 \\ \hline 0 \end{array}$$	$$\begin{array}{r} h\ t\ o \\ 1\ 8 \\ 2\,\overline{)2\ 7\ 8} \\ -2\downarrow \\ \hline 0\ 7 \end{array}$$
Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
$$\begin{array}{r} h\ t\ o \\ 1\ 3 \\ 2\,\overline{)2\ 7\ 8} \\ -2 \\ \hline 0\ 7 \end{array}$$	$$\begin{array}{r} h\ t\ o \\ 1\ 3 \\ 2\,\overline{)2\ 7\ 8} \\ -2 \\ \hline 0\ 7 \\ -\ 6 \\ \hline 1 \end{array}$$	$$\begin{array}{r} h\ t\ o \\ 1\ 3 \\ 2\,\overline{)2\ 7\ 8} \\ -2 \\ \hline 0\ 7 \\ -\ 6 \\ \hline 1\ 8 \end{array}$$
Divide 2 into 7. Place 3 into the quotient.	Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$$\begin{array}{r} h\ t\ o \\ 1\ 3\ 9 \\ 2\,\overline{)2\ 7\ 8} \\ -2 \\ \hline 0\ 7 \\ -\ 6 \\ \hline 1\ 8 \end{array}$$	$$\begin{array}{r} h\ t\ o \\ 1\ 3\ 9 \\ 2\,\overline{)2\ 7\ 8} \\ -2 \\ \hline 0\ 7 \\ -\ 6 \\ \hline 1\ 8 \\ -1\ 8 \\ \hline 0 \end{array}$$	$$\begin{array}{r} h\ t\ o \\ 1\ 3\ 9 \\ 2\,\overline{)2\ 7\ 8} \\ -2 \\ \hline 0\ 7 \\ -\ 6 \\ \hline 1\ 8 \\ -1\ 8 \\ \hline 0 \end{array}$$
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	There are no more digits to drop down. The quotient is 139.